Exploration of the QAOA* energy landscape: A case study on Maximum Independent Set

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*QAOA** :Quantum Approximate Optimization Algorithm

Context & Approach

Motivation

- Focus on mid-term quantum computers
- Impact of graph components on the QAOA energy landscape
- Discover instances and patterns beneficial to the QAOA.

Quick State of the Art

- QAOA landscape specificities:
  - Concentration of good parameters $\beta*$ $\gamma*$ [1, 2]
  - Correlation between the graph edit distance of 2 graphs and their energy landscape [3, 4]
  - The QAOA is limited by its locality [5]
  - Angles of unweighted instance seems to be reusable on weighted instances [6]
- QAOA angle optimization:
  - We use interpolation method [7] to optimize angles at p+1 from angles found at p.

Contributions

- Analysis of the behavior of QAOA solving MIS problems of weighted and unweighted graphs.
- Rescaling weights on MIS seems to work as for MaxCut.

Problem formulation

- Example of Independent set:

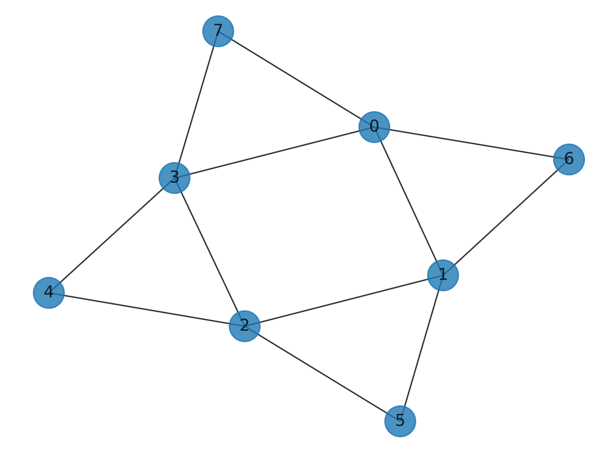

Figure 1: 8 node graph

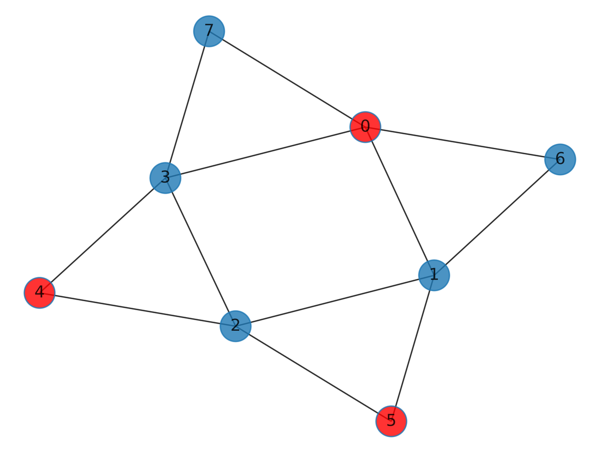

Figure 2: **Maximal** Independent Set

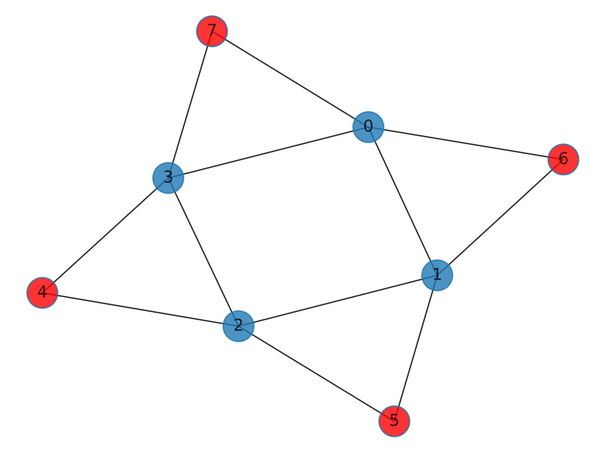

Figure 3: **Maximum** Independent set

- Given a graph $G = (V, E)$ with the set of vertices V and the set of edges E, the constraint satisfying independent set is:

$$\text{if } v \in MIS \text{ then } \forall v' \in \Gamma(v),\ x_v x_{v'} = 0 \quad (1)$$

- The cost function including the penalty term is:

$$\text{Minimize } -\sum_{v \in V'} \omega_v x_v + \sum_{v \in V'} \sum_{v' \in \Gamma(v)} \lambda_{vv'} x_v x_{v'} \quad (2)$$

$$\lambda_{vv'} = \text{Max}(\omega_v, \omega'_v) + 1 \quad (3)$$

Experiments

The QAOA

- U_P is built from the problem cost function $\gamma \in [0; 2\pi]$
- U_M defines the transition between state of the computational basis $\beta \in [0; \pi]$

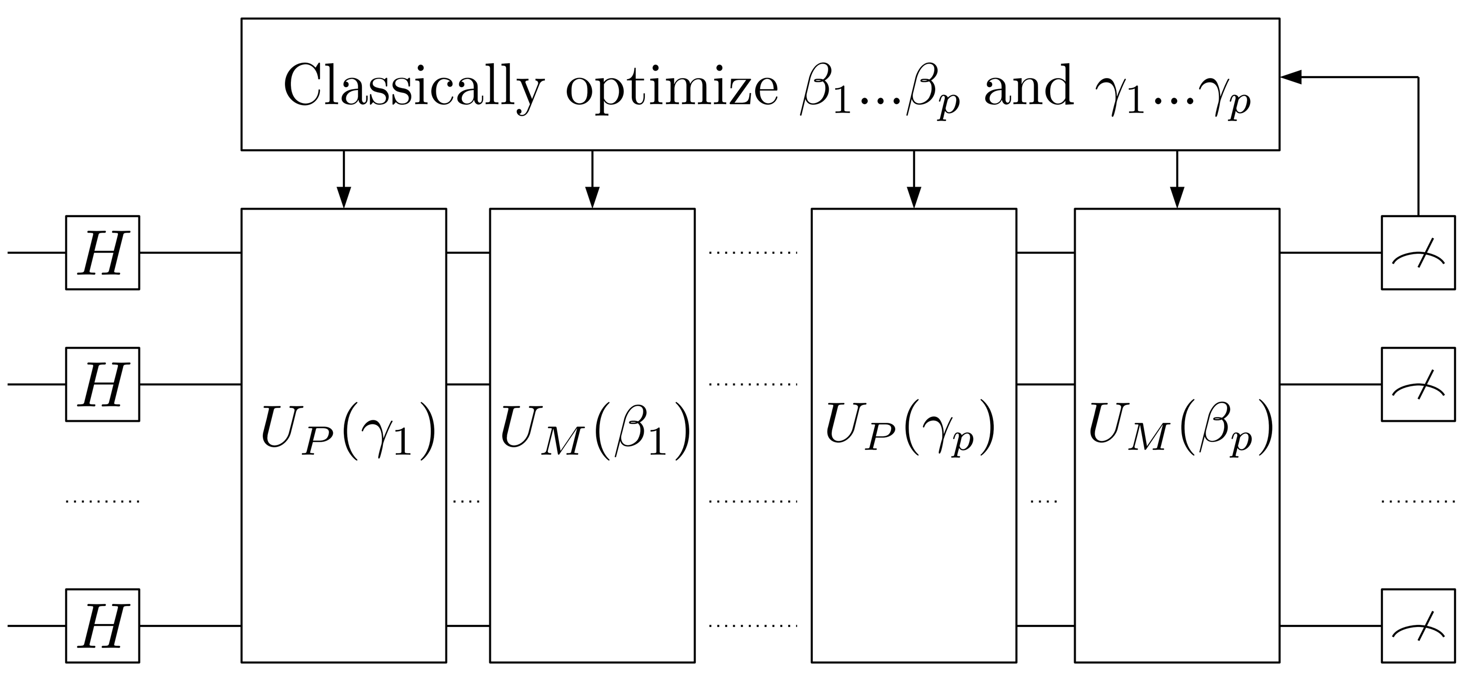

Figure 4: Schematic p-level QAOA

Impact of single node addition

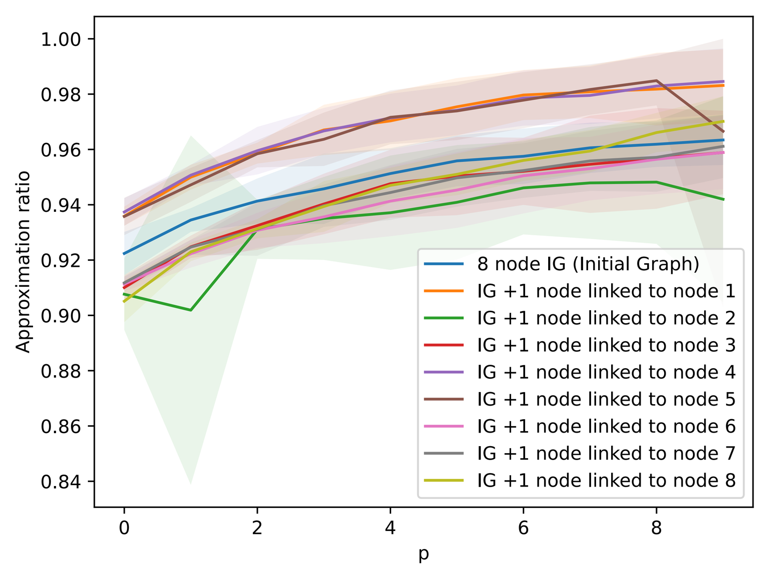

figure 5: Impact of single node addition over the Approximation ratio

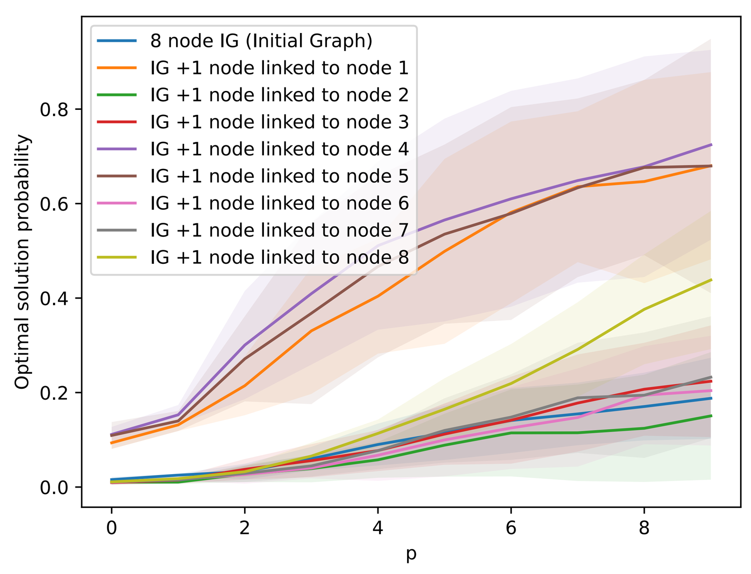

figure 6: Impact of single node addition over the Optimal solution probability

Unitaries implementation

- U_P unitary implementation:

$$U_p(\gamma):\ \omega_e Z_v \text{ terms} \Rightarrow |q_v\rangle -[R_z(2\omega_v\gamma)]-$$

$$\omega_{vv'} Z_v Z_{v'} \text{ terms} \Rightarrow |q_v\rangle, |q_{v'}\rangle: \text{CNOT} - R_z(2\omega_{vv'}\gamma) - \text{CNOT} \quad (4)$$

- U_M unitary implementation:

$$U_M(\beta):\ X_v \text{ terms} \Rightarrow |q_v\rangle -[R_x(2\beta)]- \quad (5)$$

Effect of weights on optimization landscape

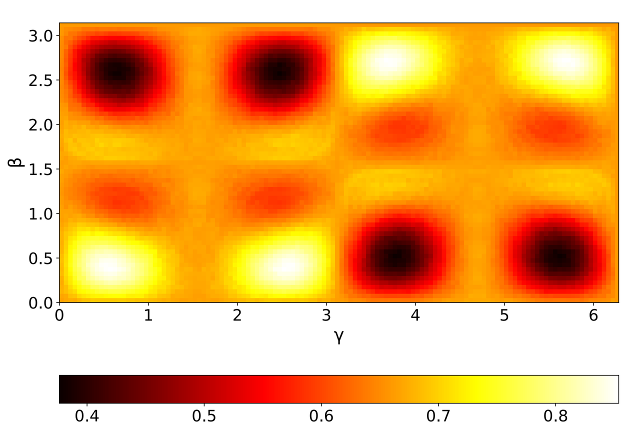

Figure 7: Unweighted

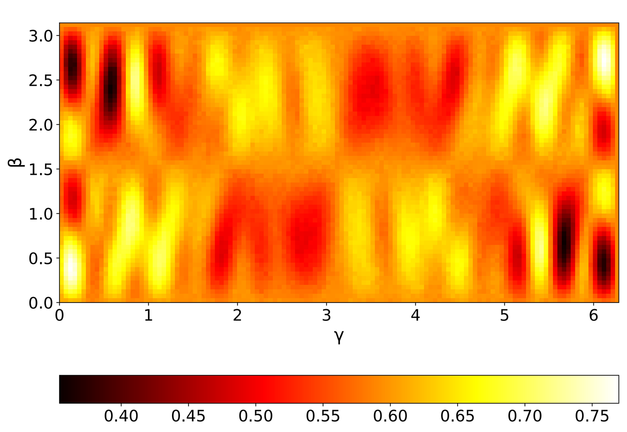

Figure 8: $w_i \in [1 - 10]$

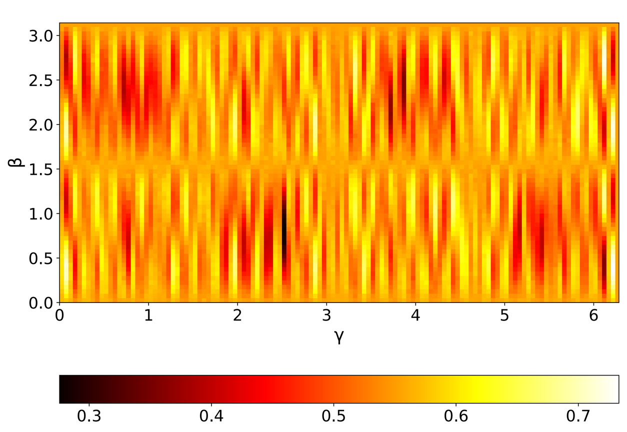

Figure 9: $w_i \in [1 - 100]$

- Comparing the optimization landscape with and without rescaling weights [6]:

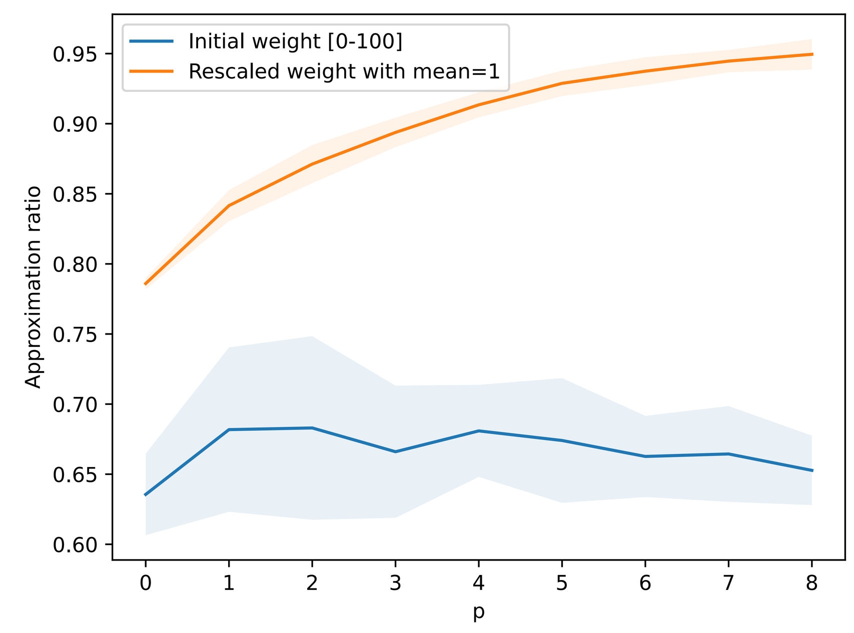

Figure 10: Single instance w / wo rescale

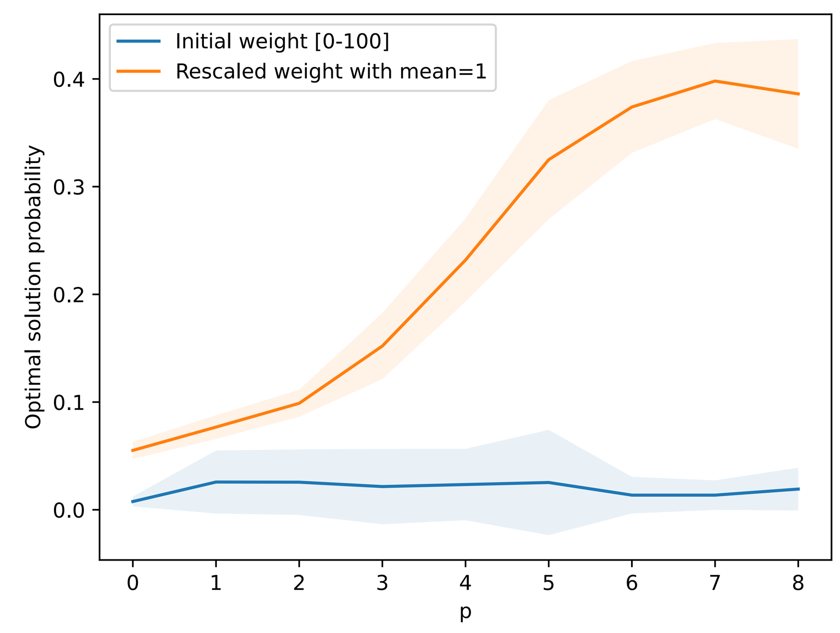

Figure 11: Single instance w / wo rescale

Future work

- Confirm results obtained with the rescaling method on numerous instances and study the case when the standard deviation is very high.
- Try to identify graph patterns favorable to the QAOA.

Bibliography

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