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SOCIÉTÉ FRANÇAISE DE RECHERCHE OPÉRATIONNELLE
ET D'AIDE À LA DÉCISION

DE LA RECHERCHE À L'INDUSTRIE

Benchmarking QAOA through Maximum Cardinality matching

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Introduction

- **Quantum computing is based on 2 main principles:**
  - Quantum superposition
  - Interference
- **In 2018 Google announced Quantum Supremacy (gate-based model):**

  "Quantum supremacy using a programmable superconducting processor" F. Arute et al.
- **Benchmark of quantum machines:**
  - Generic class of problems to study
  - Follow the evolution of quantum machines (gate fidelity and decoherence)
- **Overview:**
  - The problem
  - Introduction to variational methods
  - Benchmark of QAOA and SA
  - Conclusion

- **Maximum Cardinality Matching Problem**

  G = (V, E)

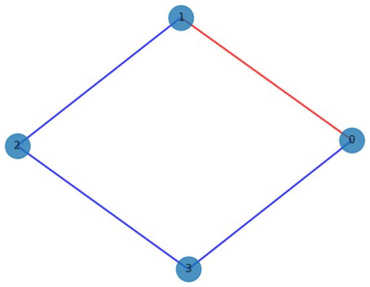
  V: set of vertices

  E: set of edges

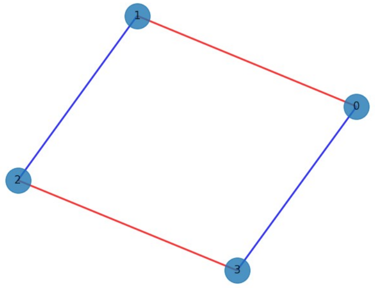
  M: set of independent edges

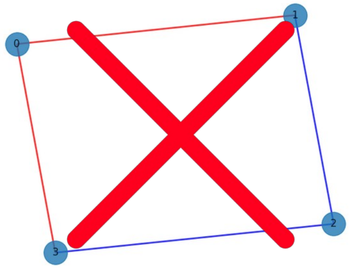
  Objective: maximize |M|

Matching



Maximum cardinality matching





- **Complexity of the problem**

	Is bipartite	Best classical complexity	Is complex for SA ?
SH graph	yes	$O(n)$	yes
Bipartite graph	yes	$O(n^{5/2})$	no (most of them)
Random graph	no	$O\left(\sqrt{\lvert V\rvert} \cdot \lvert E\rvert\right)$	no (most of them)

Implementation of the problem (Minimization form)

- **Maximization of the number of edges in the matching:**

$$\text{Minimize } -\sum_{e\in E} x_e \text{ with } x_e = \begin{cases} 1, & \text{if } e \in M \\ 0, & \text{otherwise} \end{cases}$$

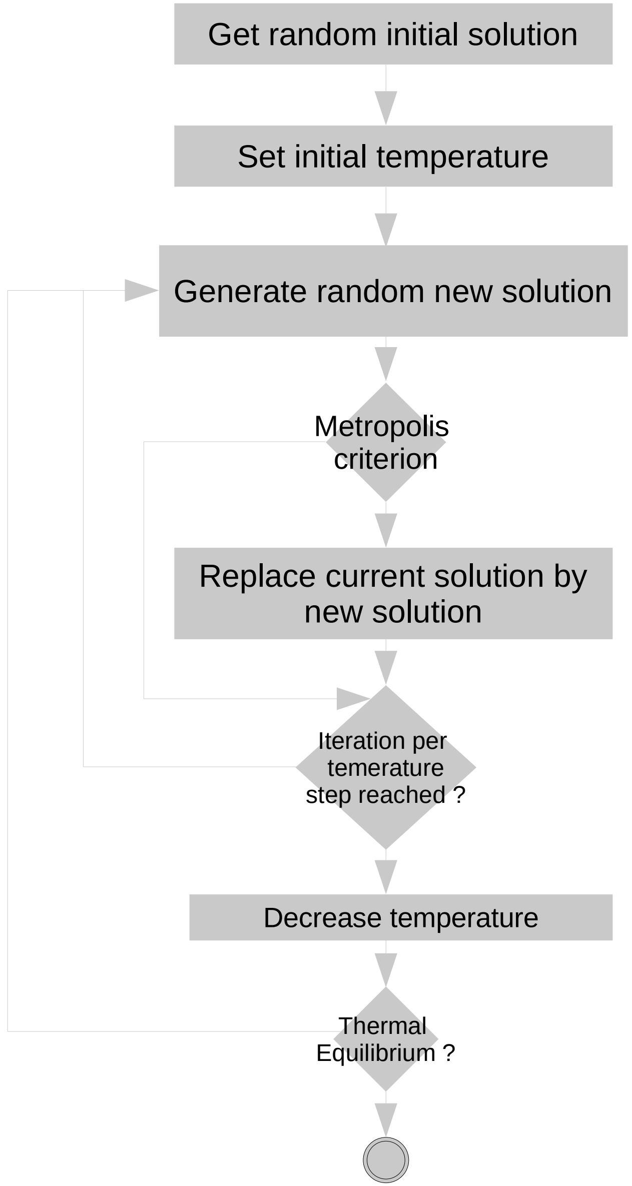
- **Constraint on independent edges:**

$$\text{if } e \in M \text{ then } \forall e' \in \Gamma(e),\ x_e x_{e'} = 0$$

- **Cost function of Maximum Cardinality Matching problem with penalty:**

$$\text{Minimize } -\sum_{e\in E} x_e + \lambda \sum_{e\in E} \sum_{e'\in\Gamma(e)} x_e x_{e'}$$

Implementation on Simulated Annealing



$$P_{accept} = e^{\frac{-\Delta_E}{T}}$$

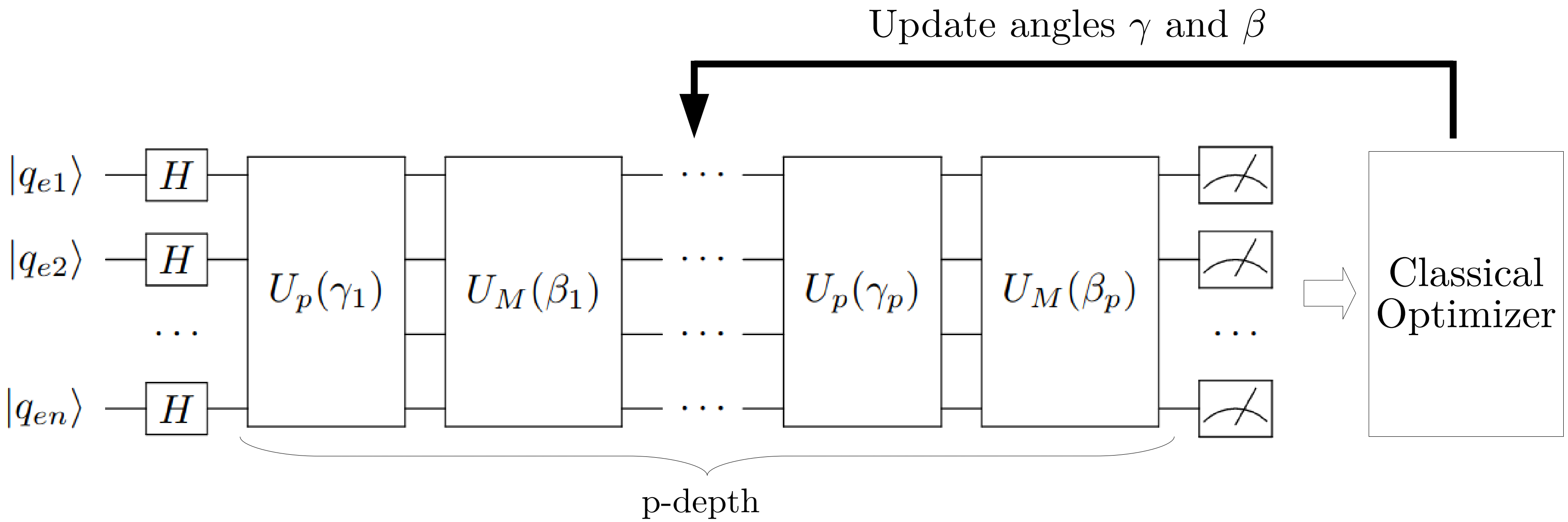
$$i <= n^x \qquad x \in \{0.2, 0.3, ...1.5\}$$

$$T_p = 0.95 * T_{p-1}$$

$$T <= 10^{-3}$$

Basic Version of QAOA [2]

- Initial state.
- Unitary operator $U_P(\gamma)$ encoding the problem based on the cost function (encoded under the Ising Model).
- Unitary operator $U_M(\beta)$ providing transition between subspace of solutions.



- **Maximum cardinality matching cost function:**

$$\text{Minimize} \quad -\sum_{e\in E} x_e + \lambda \sum_{e\in E} \sum_{e'\in\Gamma(e)} x_e x_{e'}$$

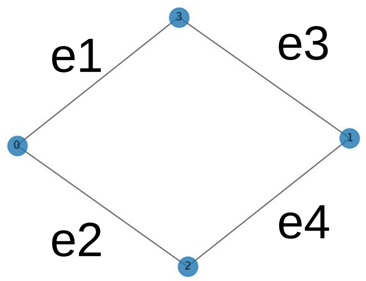
- **Implementation of (U_p) unitary encoding the Hamiltonian H_p:**

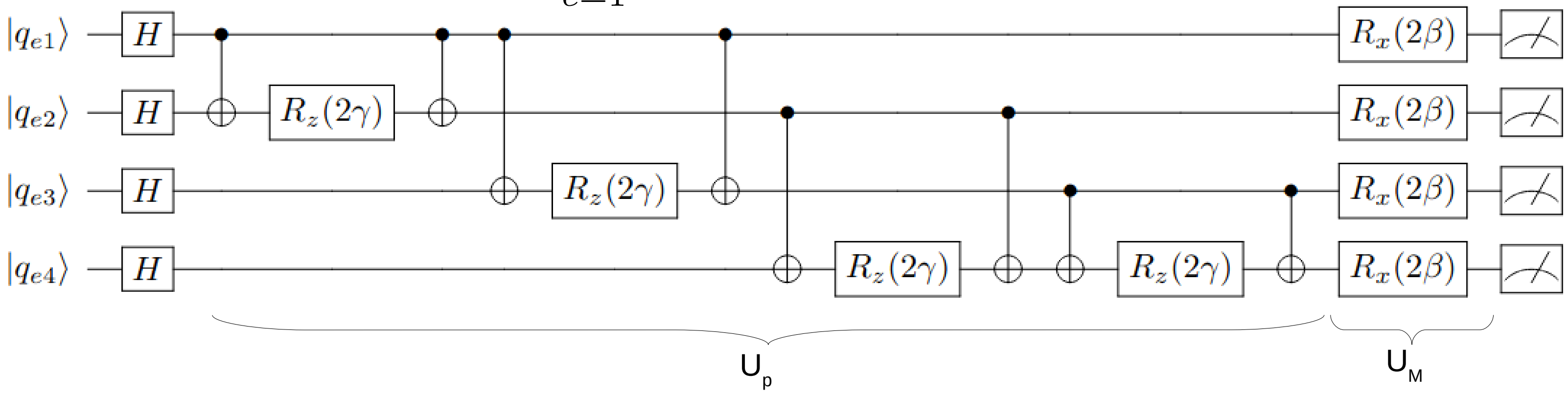
$$\mathcal{H}_P = \sum_{e}^{n} h_e \sigma_e^z + \sum_{e<e'}^{n} J_{ee'} \sigma_e^z \sigma_{e'}^z \text{ with } \sigma_e^z \text{ and } \sigma_{e'}^z \in \{-1, +1\}$$

$$x_e = (1 + \sigma_e^z)/2$$

- **Implementation of (U_M) unitary encoding the Hamiltonian H_M:**

$$\mathcal{H}_M = \sum_{e=1}^{n} \sigma_e^x$$





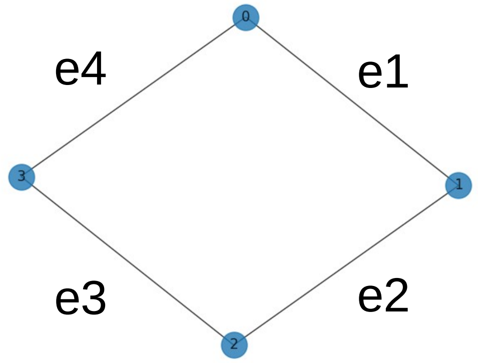
H-QAOA [1][3] an implementation reducing the search space

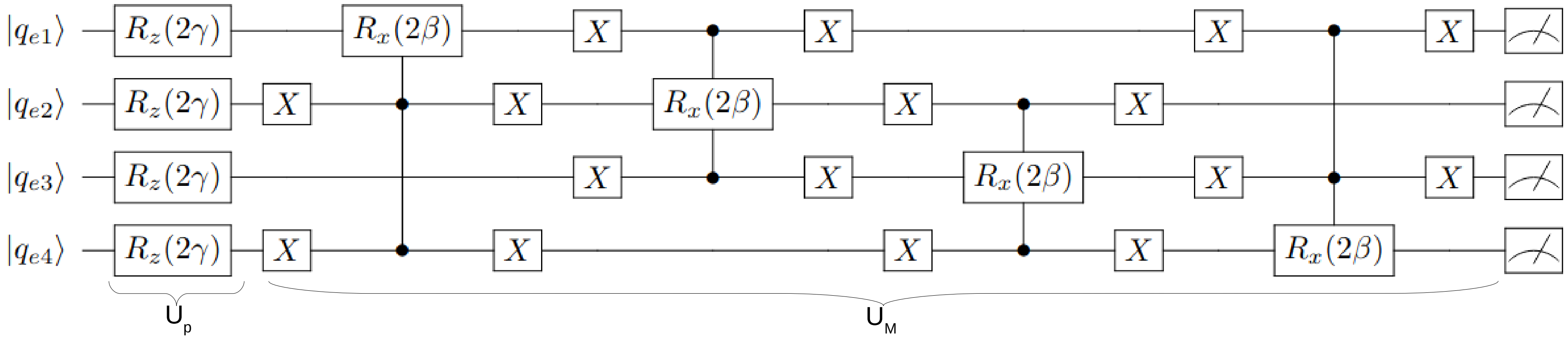
- **Principle:**
  - Remove the soft constraint from the Hamiltonian H_P.
  - Restrict the transition of states between feasible states by modifying H_M.
- **Implementation of (U_p) unitary encoding the Hamiltonian H_p:**

$$\mathcal{H}_P = \sum_{e}^{n} h_e \sigma_e^z \text{with } \sigma_e^z \in \{-1, +1\}$$

- **Implementation of (U_M) unitary encoding the Hamiltonian H_M with controlled mixers:**

$$f(e) = \prod_{e' \in \Gamma(e)} \overline{x_{e'}} \qquad \mathcal{H}_{M,e} = \bigwedge_{f(e)} \sigma_e^x$$





Benchmark metrics

- **Approximation ratio**

  E: current energy

  E_{max}: Maximum of energy (worst solution)

  E_{min}: Minimum of energy (best solution)

$$r = \frac{E - E_{max}}{E_{min} - E_{max}}$$

- **Optimal solution probability**

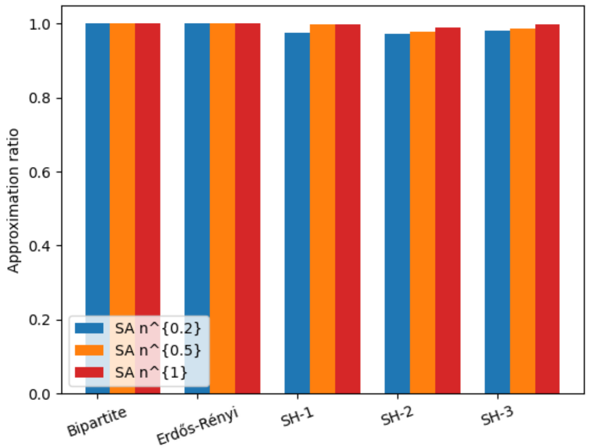
  n: amount of simulation

  z_i: bitstring

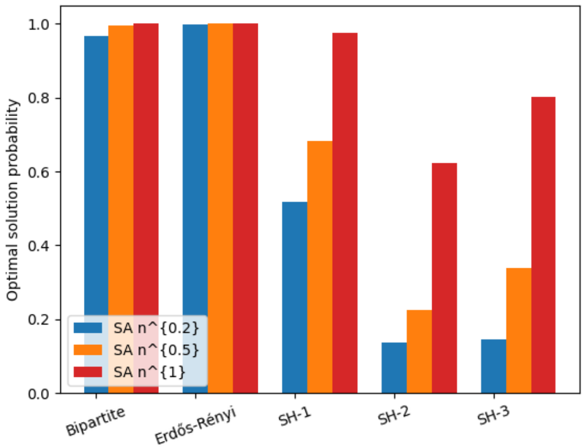
  E_{min}: Minimum of energy (best solution)

$$P_{Opt\_sol} = \frac{1}{n}\sum_{i}^{n} x_i \text{ where } x_i \begin{cases} 1 \text{ if } C(z_i) = E_{min} \\ 0 \text{ otherwise} \end{cases}$$

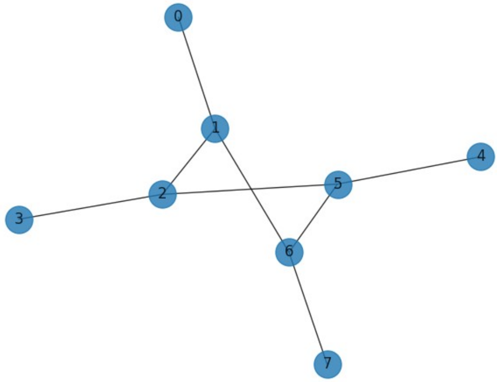
SH Graph constitutes hard instances for SA [4]



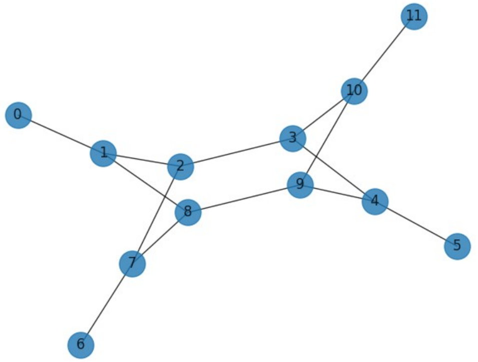
$$\lambda = 1$$



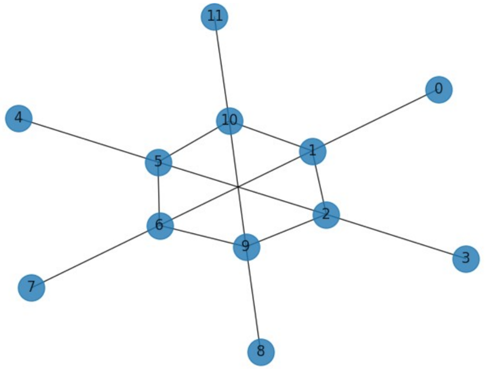
- **Study of specific instances**



SH-1

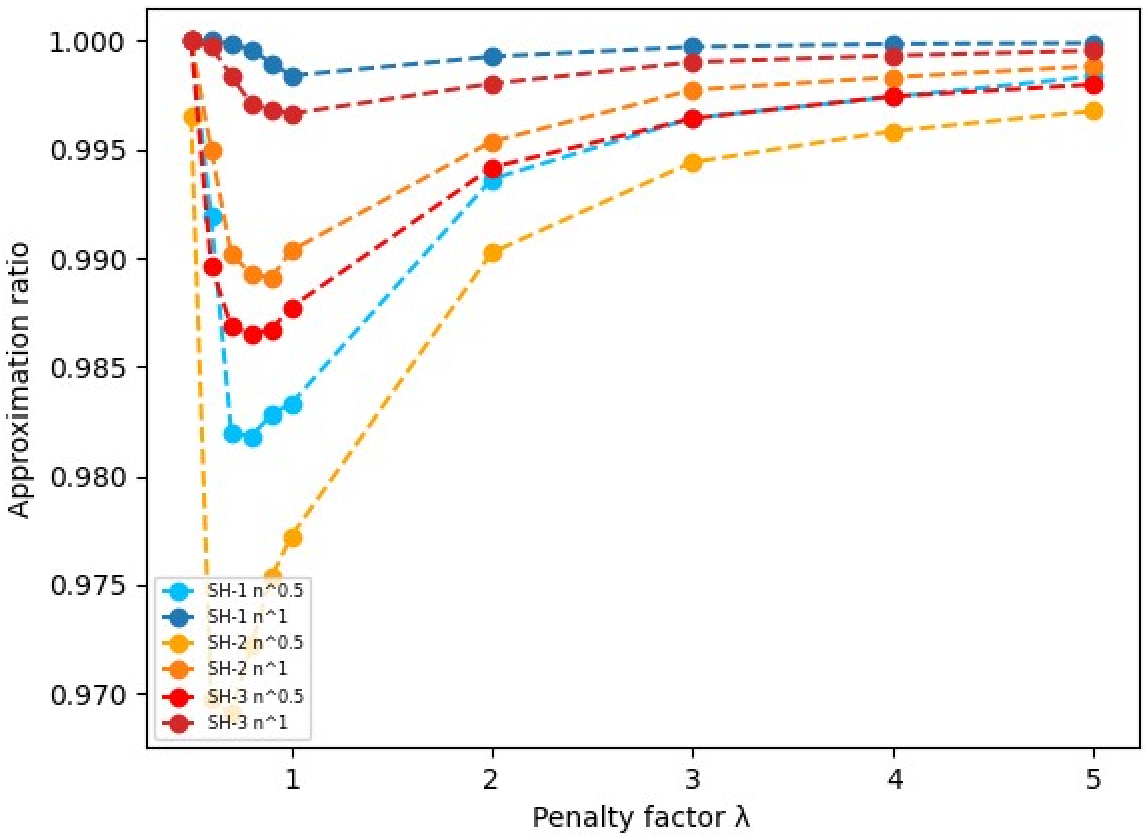


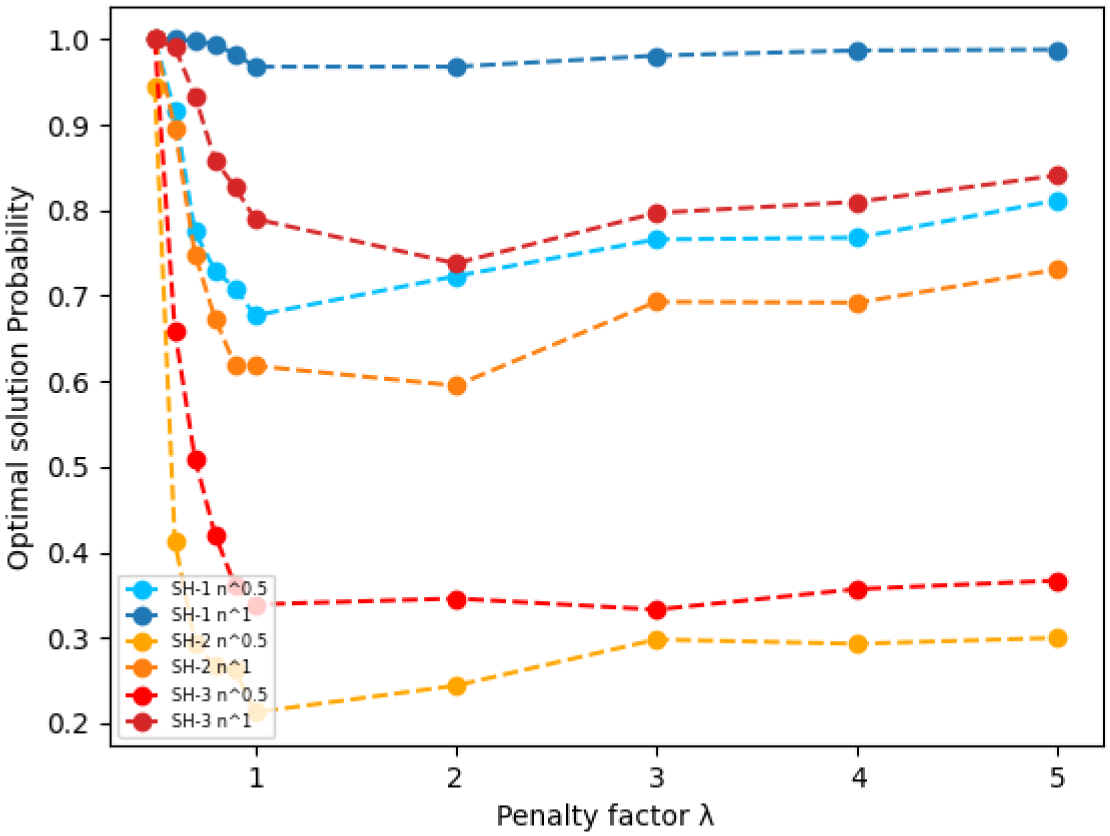
SH-2



SH-3

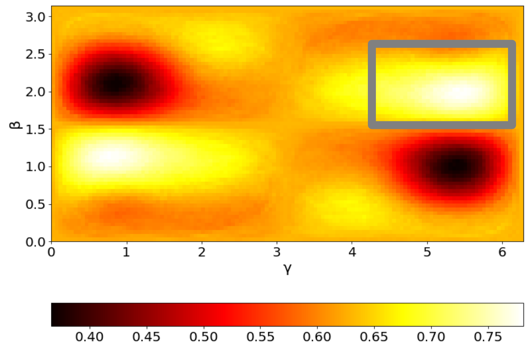
Influence of λ penalty factor over the approximation ratio and optimal solution probability



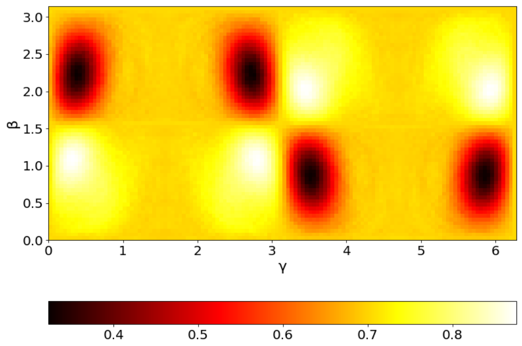


QAOA heatmap at p=1 SH-1

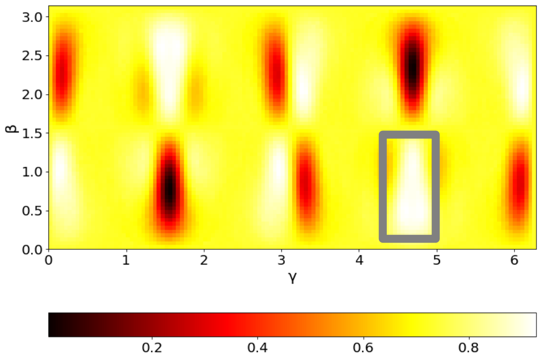
- **Approximation ratio at p=1:**



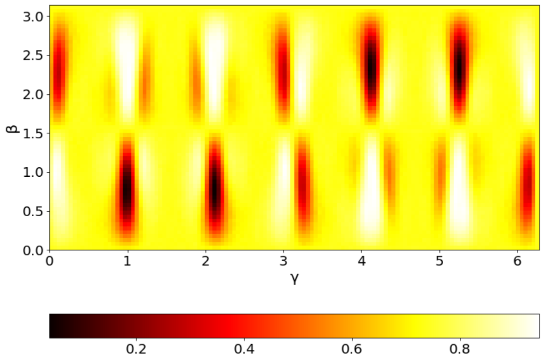
$\lambda = 0.5$



$\lambda = 1$

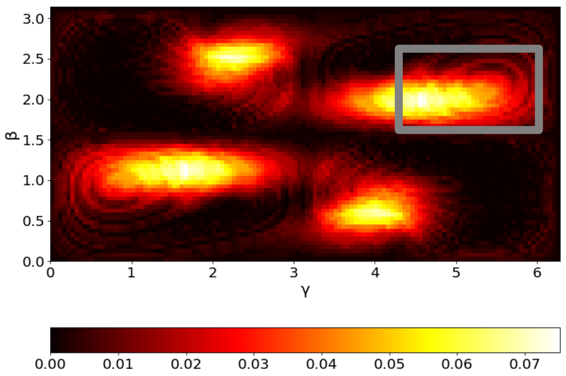


$\lambda = 2$

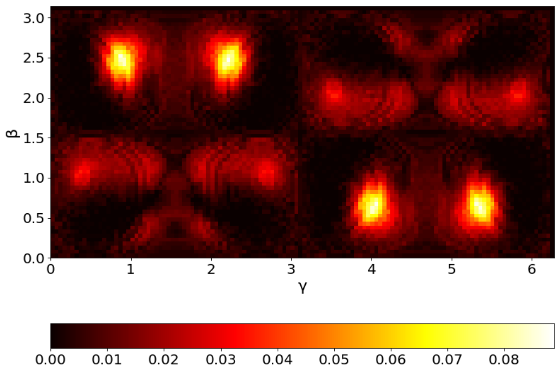


$\lambda = 3$

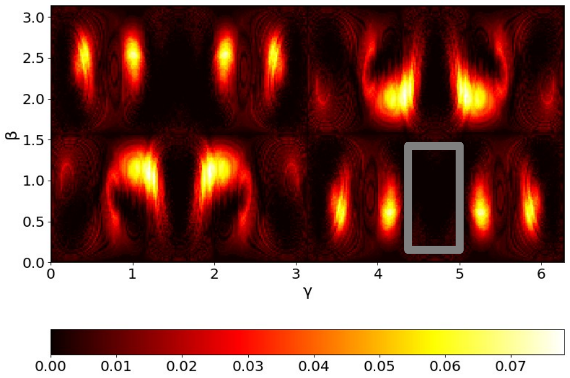
- **Optimal solution probability**



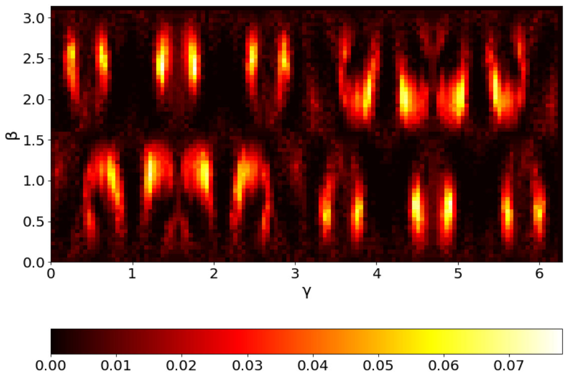
$\lambda = 0.5$



$\lambda = 1$



$\lambda = 2$

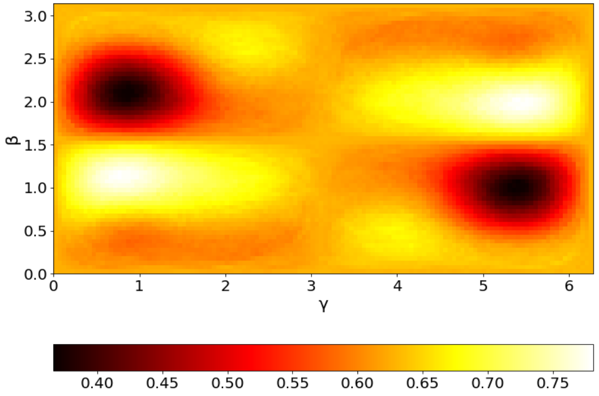


$\lambda = 3$

QAOA vs H-QAOA on SH-1

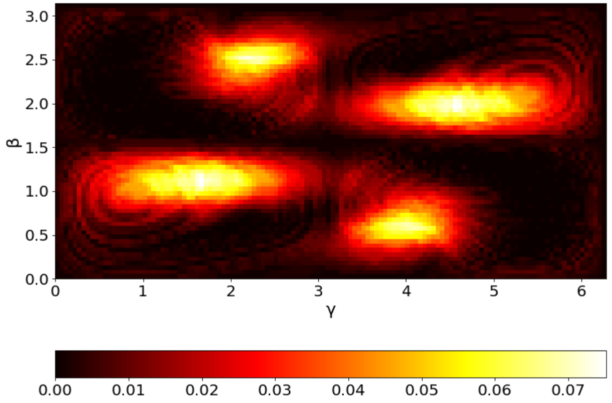
QAOA

- **Approximation ratio at p=1:**



$\lambda = 0.5$

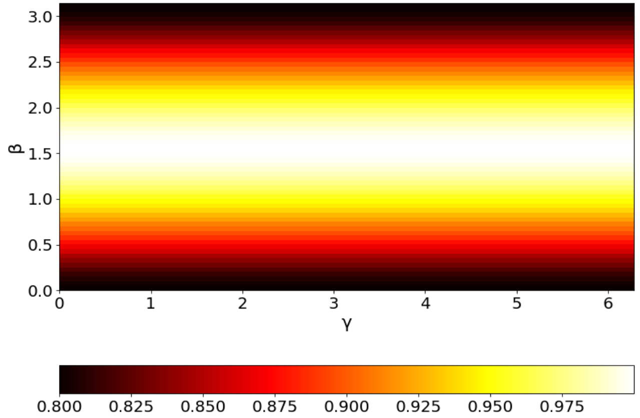
- **Optimal solution probability**



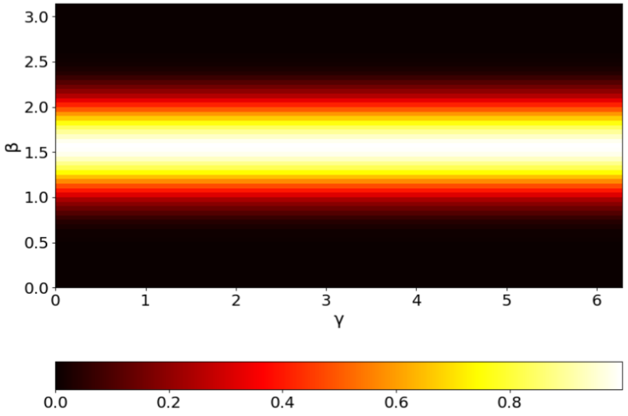
$\lambda = 0.5$

H-QAOA

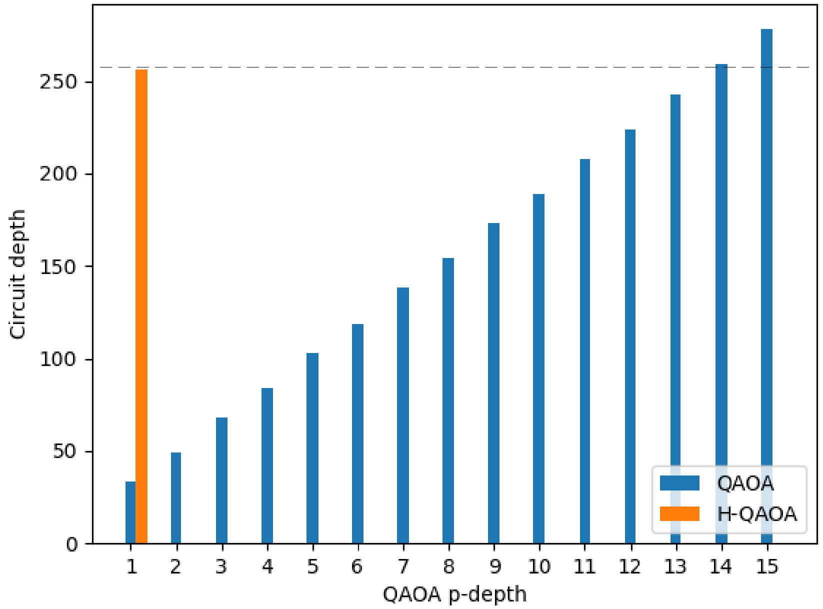
Approximation ratio at p=1:



Optimal solution probability



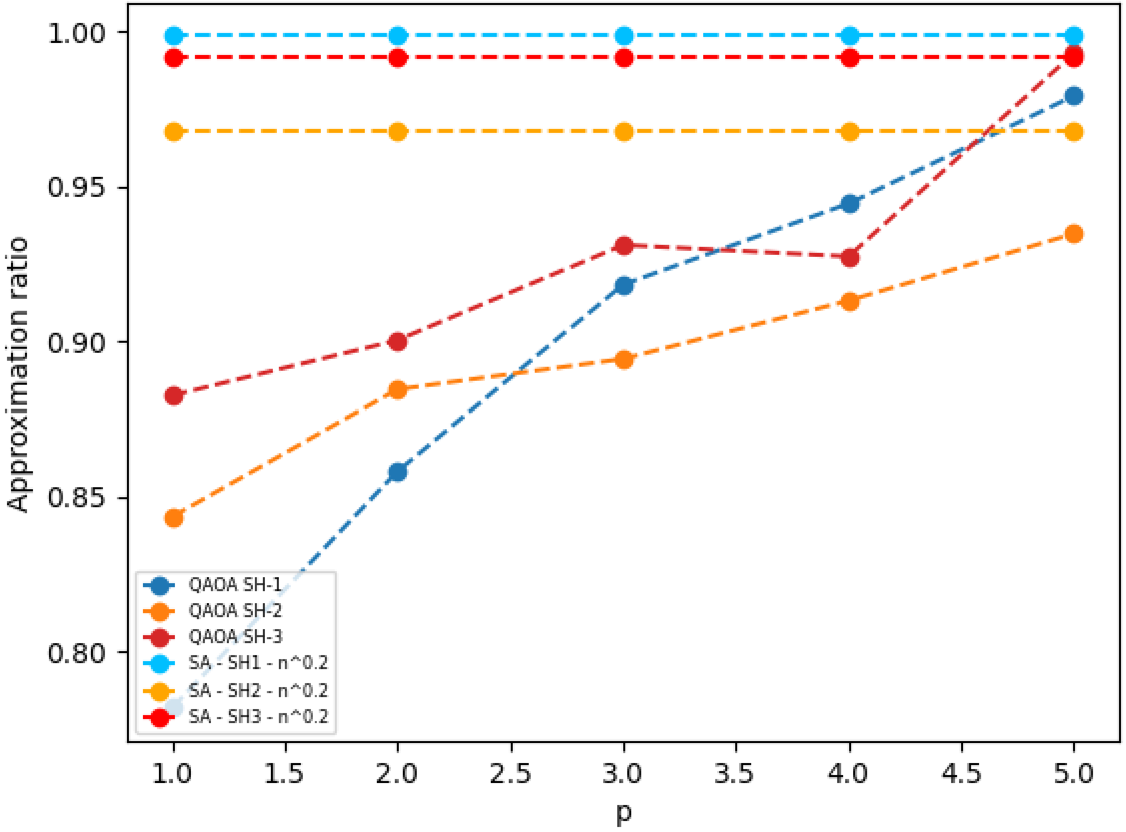
Depth comparison

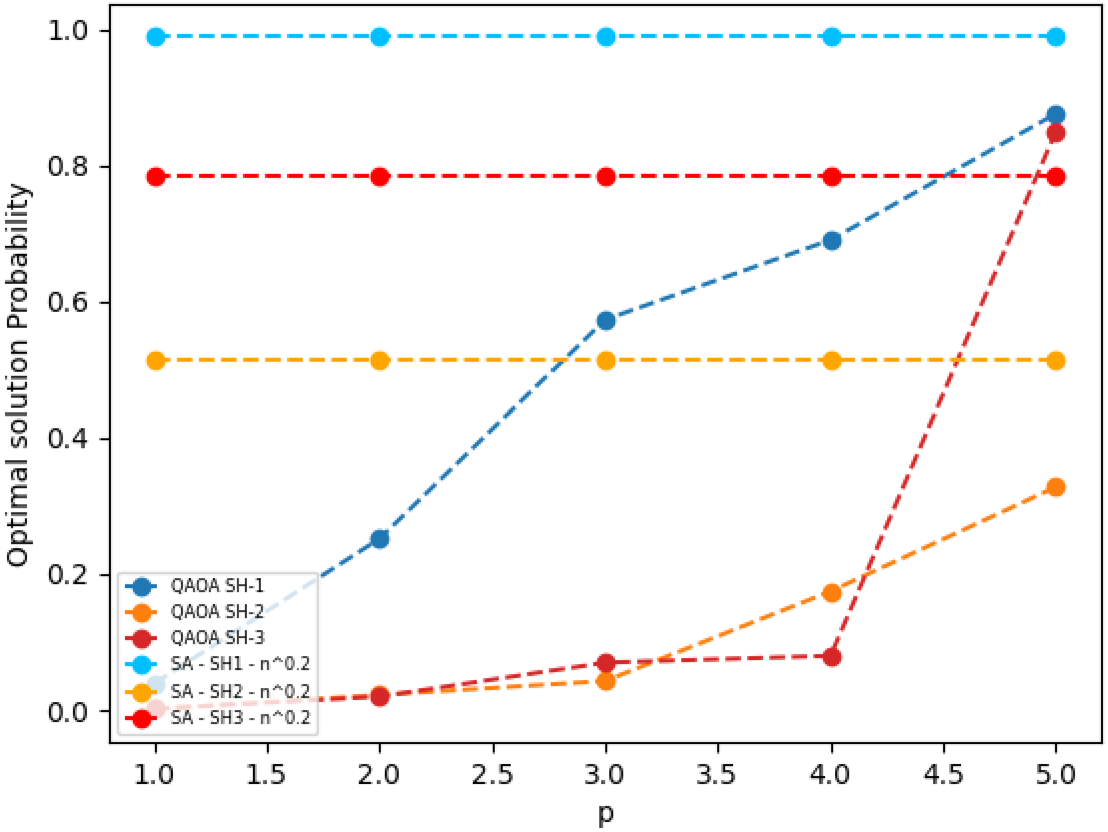


Comparison of QAOA and SA

- **Quality of the result**

$$\lambda = 0.5$$





Conclusion

- **Maximum Cardinality matching seems to be a reasonable benchmark problem**
  - QAOA seems to have similar behavior as SA
  - Polynomial problem

- **Modifying the penalty factor impact QAOA and SA**
  - Decrease the heatmap contrast
  - Increase the amount of local minima on the HeatMap

- **Limits met to benchmark the H-QAOA**
  - Size of the simulator
  - Depth impacting the time of the simulation

Bibliography

- **[1]** Sagnik Chatterjee et Debajyoti Bera. **“Applying the Quantum Alternating Operator Ansatz to the Graph Matching Problem”**. 2020. eprint : arXiv:2011.11918.
- **[2]** Edward Farhi, Jeffrey Goldstone et Sam Gutmann. **“A Quantum Approximate Optimization Algorithm”** 2014. eprint : arXiv:1411.4028.
- **[3]** Stuart Hadfield et al. **“From the Quantum Approximate Optimization Algorithm to a Quantum Alternating Operator Ansatz”**. In : 12.2 (fév. 2019), p. 34. doi : 10.3390/a12020034. url : https://doi.org/10.3390/a12020034.
- **[4]** Galen H. Sasaki et Bruce Hajek. **“The time complexity of maximum matching by simulated annealing”.** In : 35.2 (avr. 1988), p. 387-403. doi : 10.1145/42282.46160. url :https://doi.org/10.1145/42282.46160.
- **[5]** Daniel Vert. **“Étude des performances des machines à recuit quantique pour la résolution de problèmes combinatoires”.** 2021UPASG026. Thèse de doct. 2021. http://www.theses.fr/2021UPASG026/document

Merci de votre attention