Benchmarking Quantum Annealers with Near-Optimal Minor-Embedded Instances

Valentin Gilbert[1] - Julien Rodriguez[2] - Stéphane Louise[1]

[1]Université Paris-Saclay, CEA, List, F-91120, FRANCE

[2]Université de Montpellier LIRMM, CNRS, FRANCE

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I- Context & Motivations

- Different approaches to benchmark quantum computers:
  - **Component**-level benchmarks (e.g. state tomography [1], operation fidelity assessment [2])
  - **System**-level benchmarks (e.g. QV [3], Cycle benchmarking [4] ...)
  - **Application**-level benchmarks (e.g. QED-C [5], Q-score [6] ….)

- Benchmarking Quantum Computers is difficult:
  - Quantum Computers are **hybrid** by design. The specification of each process requires transparency:
    - **Compilation process** and pre-processing time
    - Quantum Computer **parameter settings**
    - **Error mitigation** methods

Do not compile away the complexity of the problem

I- Context & Motivations

- Classification of model-independent benchmark instances A. Abbas et al. [12]

Application benchmark instances

Random instances :
+ Generate large sets of instances
- Relevance of the set can be debated
- Phase transition issue for some instances

Crafted instances :
+ Easy to find instances hard for classical solvers
- Some classical methods may take advantage of the structure of the instance

Real-world instances :
+ Ideal targets
- Hard to design due to quantum hardware dimensionality
- Hard to find by nature (selection bias)

Source	Instance type	Instance topology	#Variables	QPU	Embedding
2021 [13]	BFM, FBFM, CBFM	Chimera graph	2032	2000Q	QPU chip sub-graph
2022 [14]	CBFM-P	Pegasus graph	5387	Adv4.1	QPU chip-subgraph
2023 [15]	Unweighted max-cut	3-regular graph	4-320	Adv4.1	QPU chip-subgraph
2024 [16]	2D & 3D ising sping glass	Square, cubic, Diamond, biclique	16-567	Adv4.1 Adv2	QPU chip-subgraph 2qubits/var

I- Context & Motivations

- Aim of our benchmark protocol (adapted to D-Wave QA [7]):
  - Evaluate the performance of Quantum Annealers on instances of gradual difficulty (**near-optimally embedded**).
  - Find **classes of optimization problems** for which QA might be efficient.
  - Evaluate the performance of QA according to its major bottleneck, i.e., **matrix density.**
  - Build *crafted instances* of increasing difficulty (in terms of density)

- Our protocol helps to:
  - Identify classes of optimization problem for which QA may be efficient
  - Evaluate QA performance in an ideal compilation case (best-case behavior)
  - Compare the performance of QA with classical algorithms

- Our protocol does not help to:
  - Assert that QA > classical heuristics
  - Fairly compare different quantum computers performance
  - Finely characterize the source of QA performance

II- Background – An optimization problem on QA

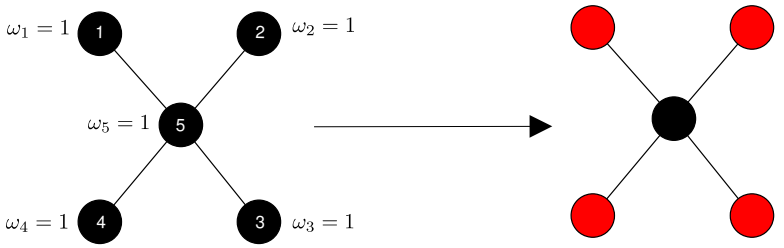
- D-Wave QA Hamiltonian

$$H(s) = A(s)H_{\mathrm{M}} + B(s)H_{\mathrm{P}}$$

$$H_{\mathrm{M}} = \sum_{v \in V_{\mathrm{s}}} \sigma_v^x$$

$$H_{\mathrm{P}} = \sum_{v \in V_{\mathrm{s}}} h_v \sigma_v^z + \sum_{(u,v) \in E_{\mathrm{s}}} J_{uv} \sigma_u^z \sigma_v^z$$

- Maximum independent set problem



- Cost function

Constraint released in the cost function

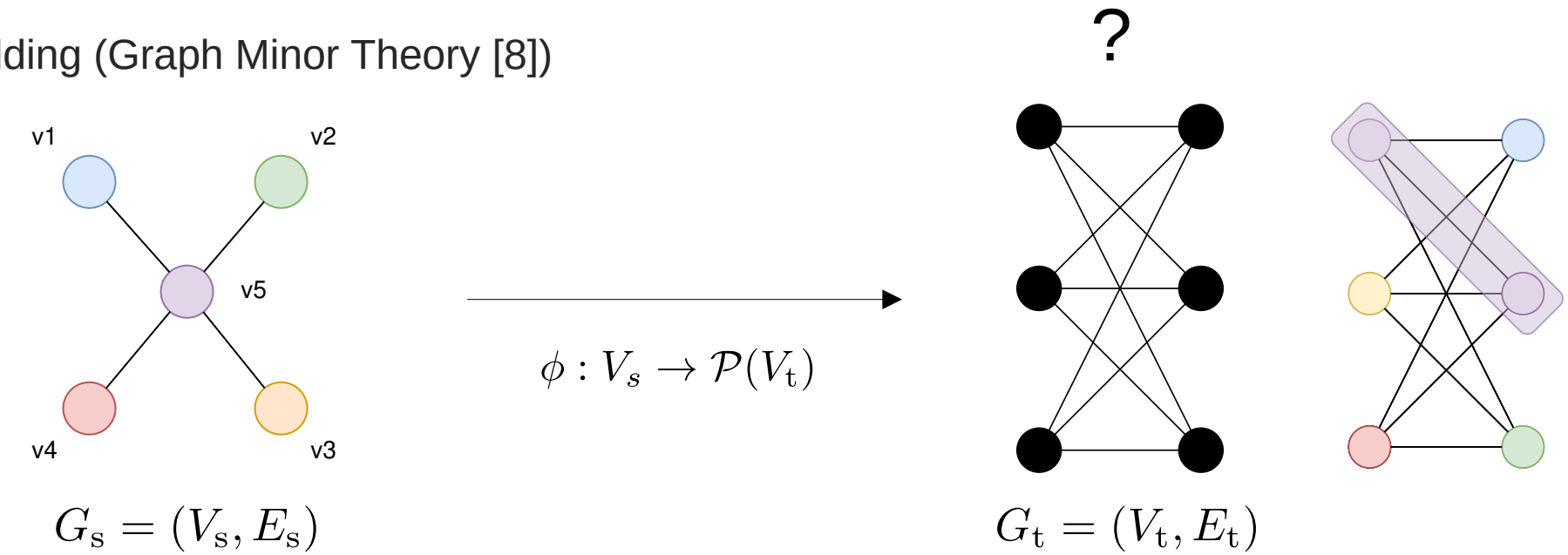
$$\min C(\mathbf{x}) = -\sum_{v \in V_{\mathrm{s}}'} \omega_v x_v + \overbrace{2 \sum_{(u,v) \in E_{\mathrm{s}}'} x_u x_v} \qquad x_i \in \{0,1\}$$

Substitute variables $x_i = \frac{1 - s_i}{2}$ to build the Ising cost function $C(\mathbf{s})$

The ground sate of H_{P} encodes the solution to the problem $\min C(\mathbf{s})$

II- Background - Minor-embedding

- Minor-embedding (Graph Minor Theory [8])

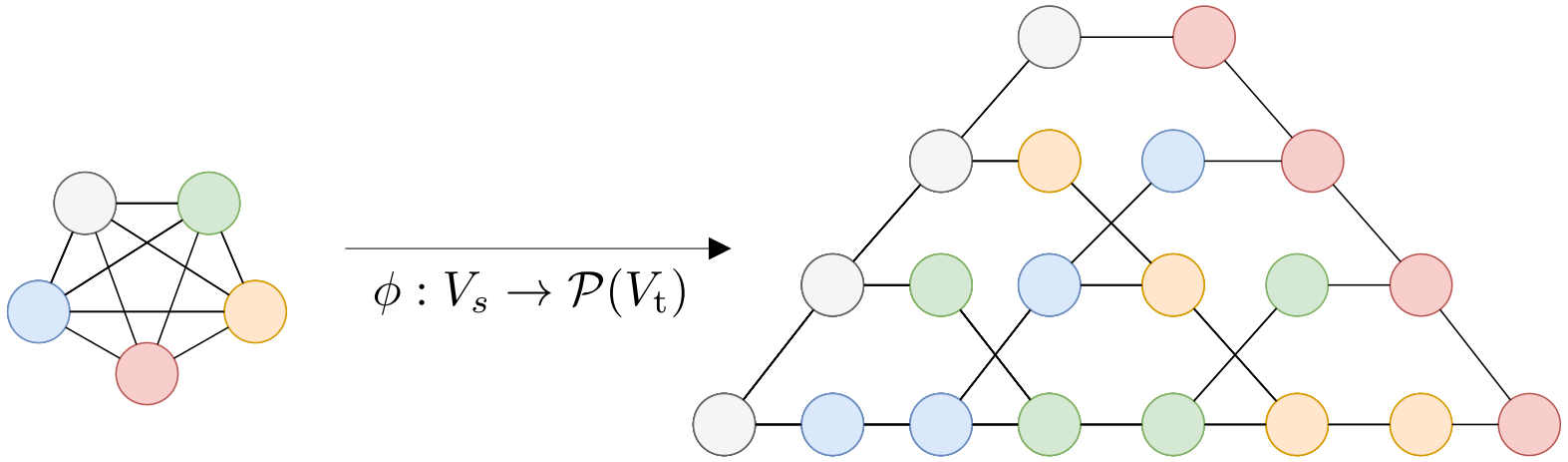


- Set of rules:

1. Each vertex $v \in V_s$ is mapped onto a connected subgraph $\phi(v)$ of G_t
2. Each connected subgraph must be vertex disjoint: $\phi(v) \cap \phi(v') = \emptyset$ for $v \neq v'$
3. $\forall (u, v) \in E_s, \exists u' \in \phi(u), \exists v' \in \phi(v)$ such that $(u', v') \in E_t$

II- Background

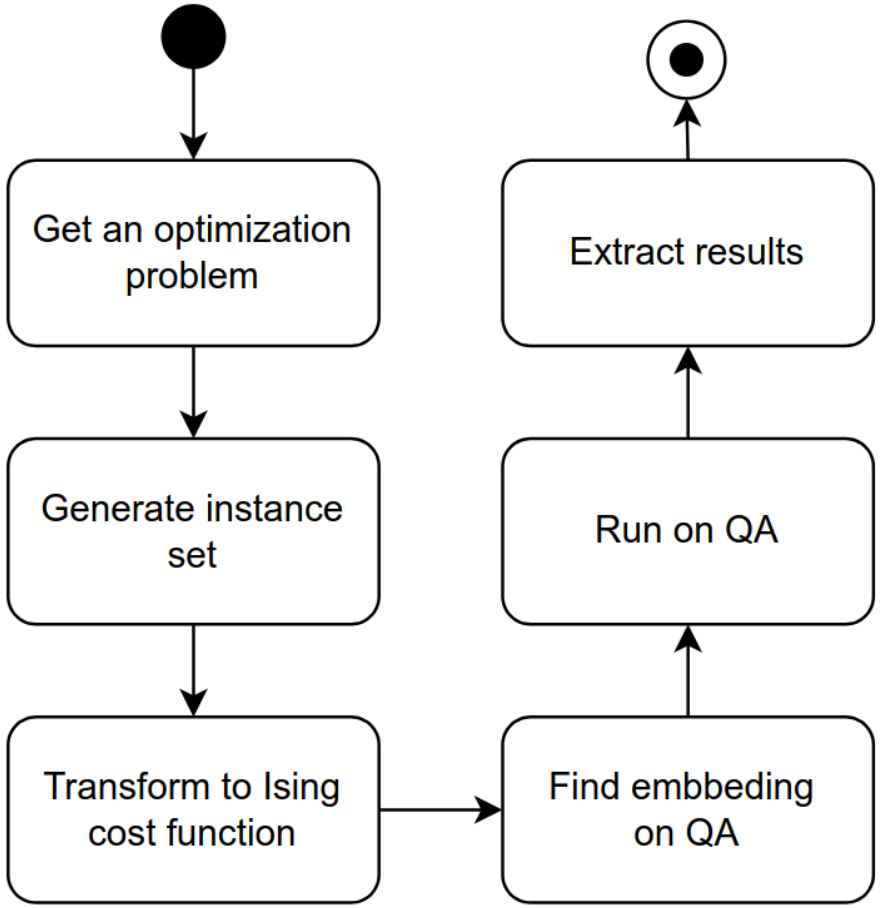
- Minor-embedding Method
  - Near-Optimal CME (Clique Minor Embedding [9] for specific target graphs (Example with the TRIAD Pattern [10])



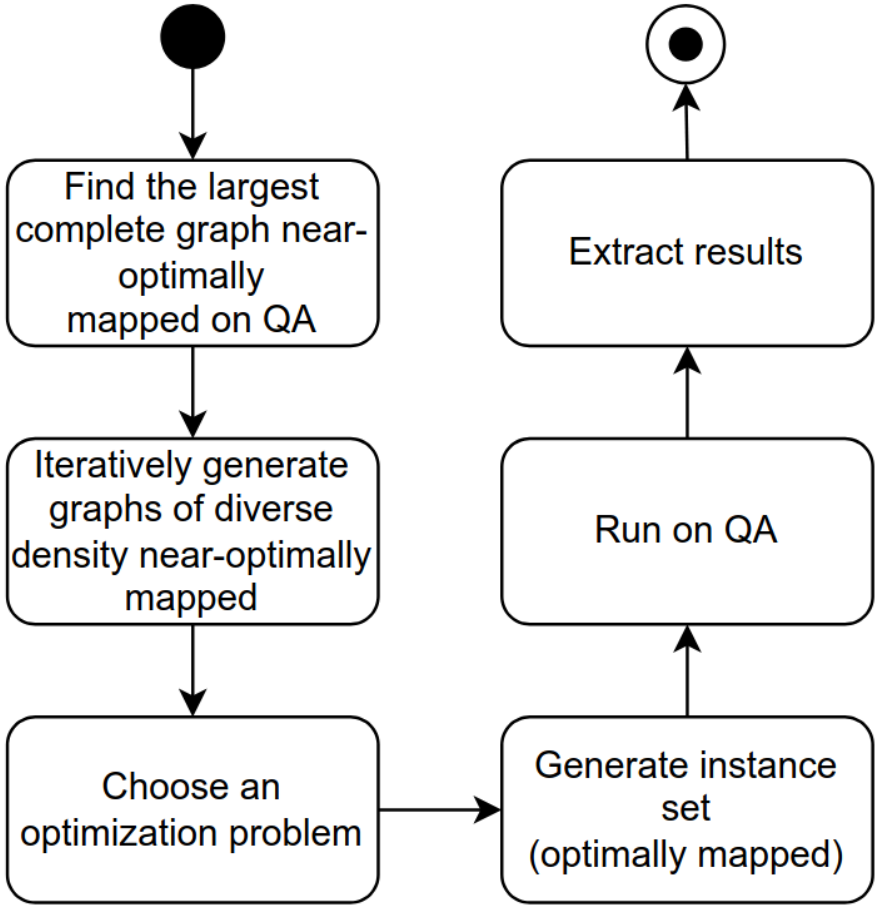
- Impact of the minor-embedding:
  - Add extra qubits
  - Changes the problem Hamiltonian (impact on the quantum evolution and the minimum spectral gap)
  - Require the setting of the so-called « chain strength » (maintains the ferromagnetic bound).
  - Require extra-processing (e.g. majority voting)

III- Method - Workflow

1) Classical benchmarking approach



2) Our approach (generation of the **crafted instances**)



III- Method – Design of the crafted instances

- How to assess the quality of a mapping:
  - D-Wave QA has a regular topology

    For Pegasus topology: $c_{\text{phys}} = 15$

  - Definition of a lower bound on the number of physical qubits used in a mapping:

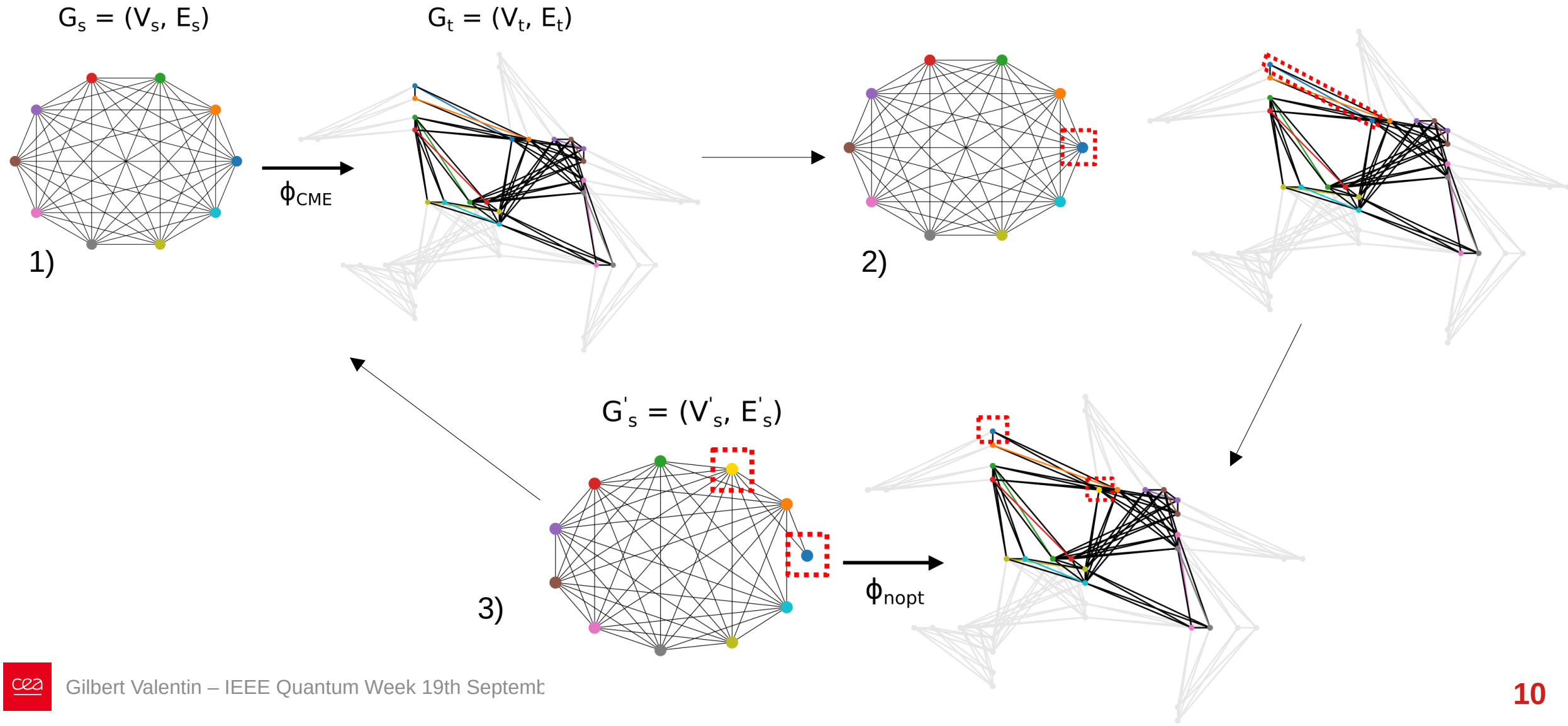
$$n_{\phi(v)^*} = \begin{cases} 1 \text{ if } deg(v) \leq c_{\text{phys}} \\ 2 \text{ if } c_{\text{phys}} < deg(v) \leq (2c_{\text{phys}} - 2) \\ \left\lceil \frac{deg(v) - (2c_{\text{phys}} - 2)}{c_{\text{phys}} - 2} \right\rceil + 2 \text{ otherwise} \end{cases}$$

  - Compute the overhead ratio considering this bound:

$$r_{\text{o}} = \frac{n_\phi}{\sum_{v \in V_s} n_{\phi(v)^*}}$$

III- Method – Design of the crafted instances

- Iterative split method for the creation of near-optimally mapped instances



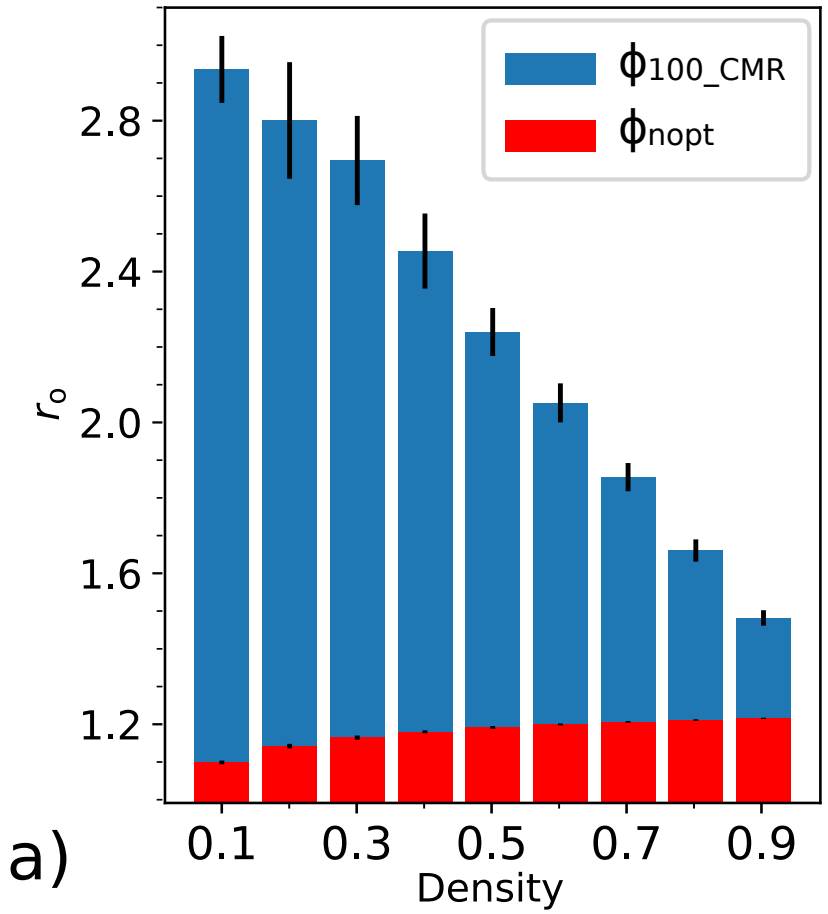
III- Method – Design of the crafted instances

- Comparison of the performance of our generation method against state of the art embedding method

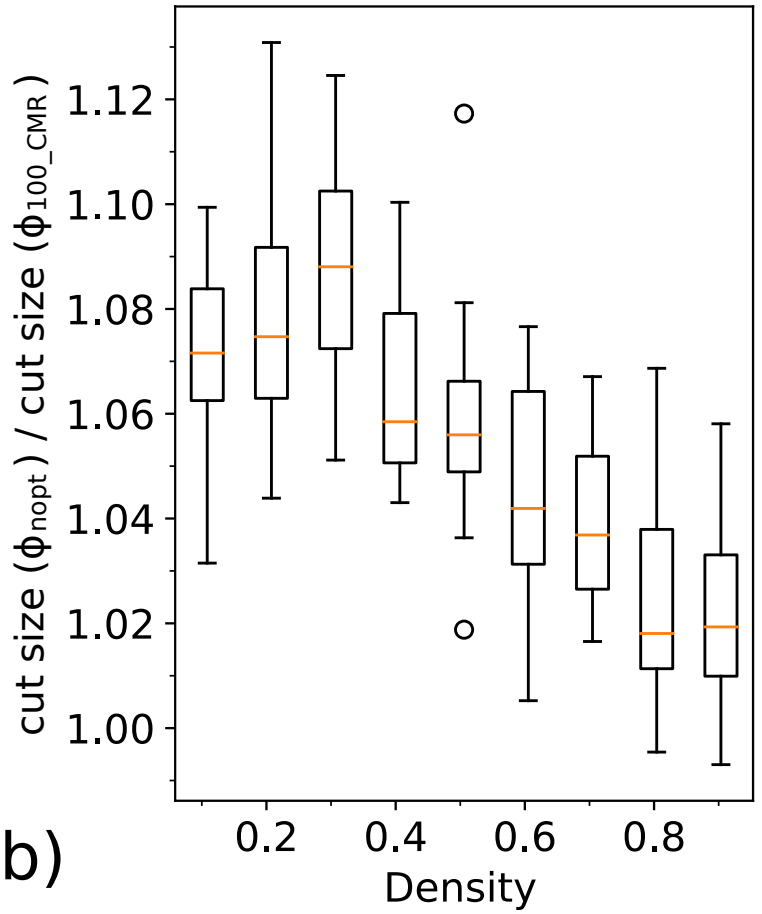
at $d = 1$

$|V_s| = 100$

$|V_t| = 982$



a)



b)

Assumption: Instances with less duplicated qubits are more easily solved by QA => Seems to be true

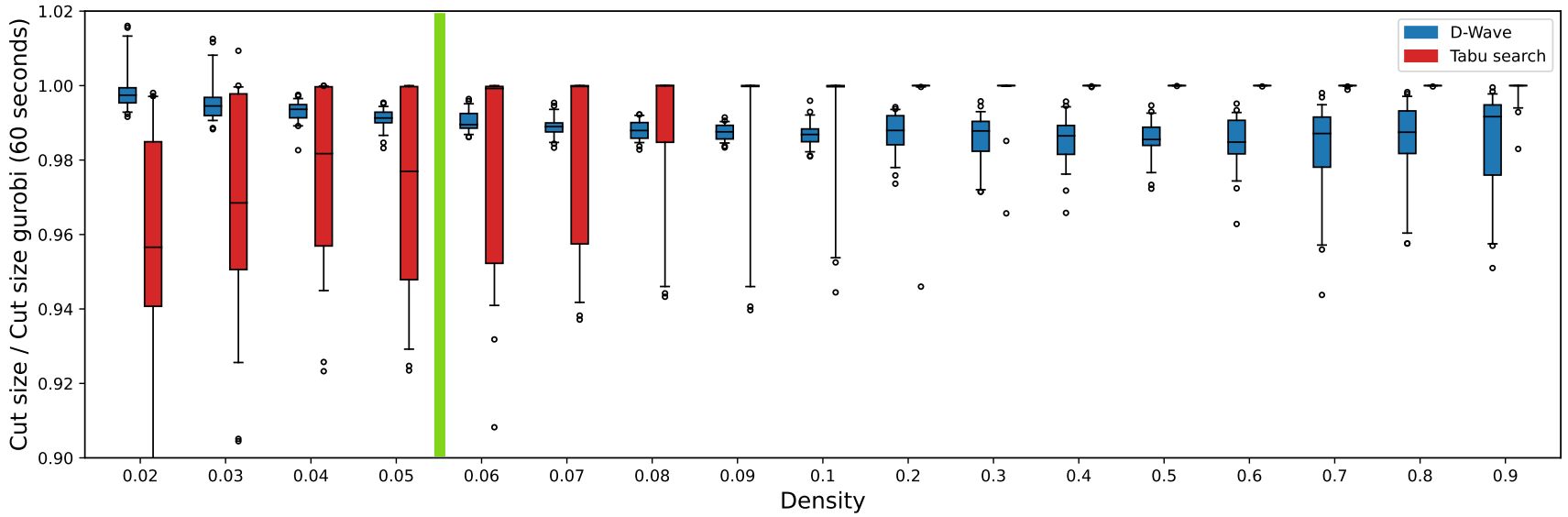
IV- Results

Time windows:
60s: Gurobi
1s: Tabu Search
1s: D-Wave

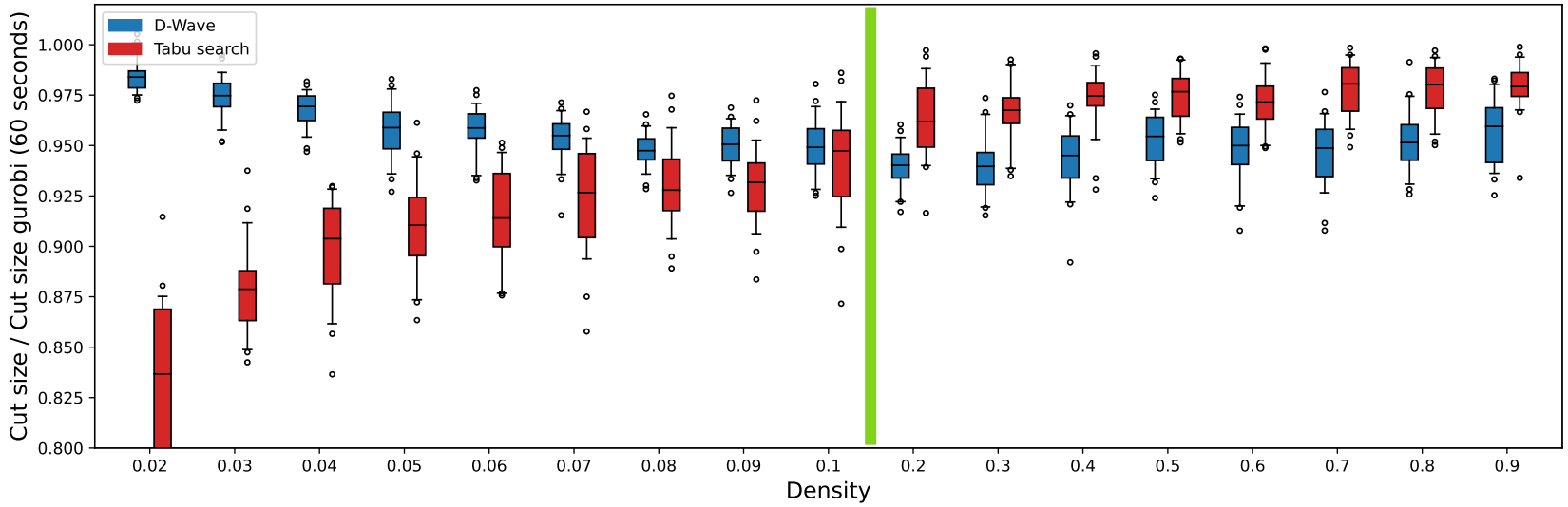
Density	Avg $\lvert V'_s \rvert$	r_o
0.02	1318	1.06
0.03	1062	1.08
0.04	912	1.10
0.05	810	1.11
0.06	737	1.12
0.07	680	1.12
0.08	635	1.13
0.09	597	1.13
0.1	565	1.14
0.2	395	1.16
0.3	321	1.17
0.4	277	1.18
0.5	248	1.19
0.6	226	1.19
0.7	209	1.19
0.8	195	1.20
0.9	184	1.20

$|V_t| = 2918$

- Unweighted Max-cut



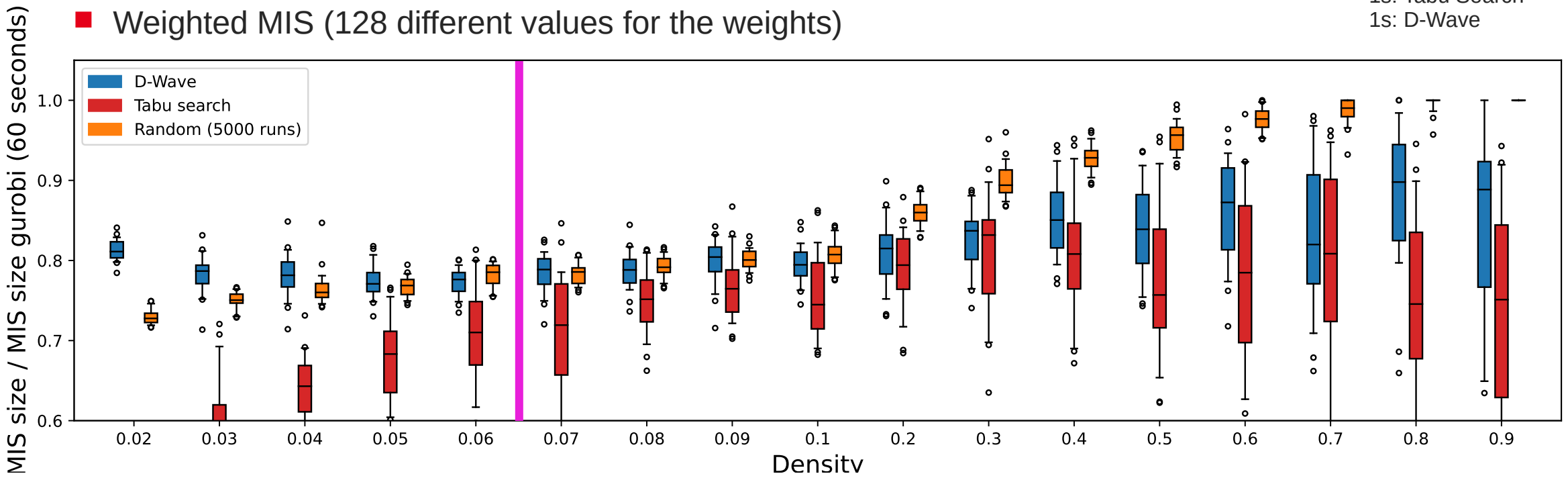
- Weighted Max-cut (256 different values for the weights)



IV- Results

Time windows:
60s: Gurobi
1s: Tabu Search
1s: D-Wave

- Weighted MIS (128 different values for the weights)



Performance intersection with random greedy search

Density	Avg $\lvert V'_s \rvert$	r_o	Density	Avg $\lvert V'_s \rvert$	r_o
0.02	1318	1.06	0.1	565	1.14
0.03	1062	1.08	0.2	395	1.16
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0.09	597	1.13	0.8	195	1.20
			0.9	184	1.20

$$|V_t| = 2918$$

V- Discussion - Perspectives

- This study suggests that:
  - At a fixed physical resources (#qubits), the QA seems to perform well for problems that are sparse (under 0.1 of density with number of logical variables > 550).

    => it limits the use of QA for dense optimization problems (such as TSP).
  - Soft constraints drastically penalize the QA and Tabu Search.

- The performance results should be considered cautiously:
  - The generated instances are very favorable to D-Wave QA (near-optimally embedded).
  - Both QA and classical algorithms can be further tuned.

  The aim of the benchmark is to **identify when** the **QA performs well** with **fixed hardware resources**.

- Future perspectives
  - Analyze how pre and post-processing methods impact the average quality of the result in this frame.
  - Extend the approach to the QAOA (with optimized planted swapping networks).
  - Identify optimization problems for which the QA performs well.

VI- Bibliography

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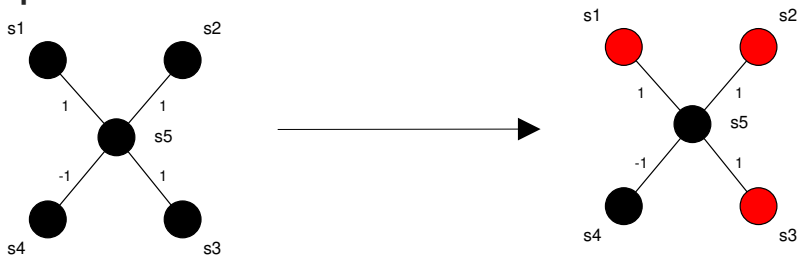
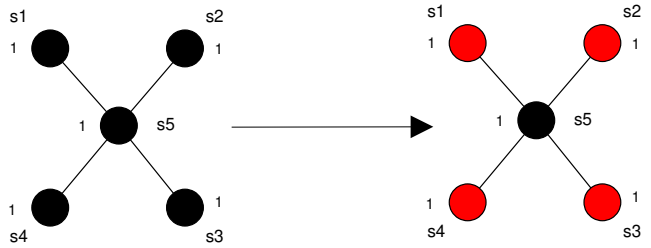
III- Method - Metrics to measure

- Quality metrics are problem-dependent
  - Max-cut problem: The cut size



cut-size = 3

  - Maximum Independent set problem: The size of the independent set.



IS size = 4

- Time measurement
  - 60s time window for branch & bound algorithm
  - 1s time window for Tabu Search (C implementation)
  - 1s time window for D-Wave Q