Quantum Annealers Chain Strengths: A Simple Heuristic to Set Them All

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I- Context & Motivations – Quantum Annealing (QA)

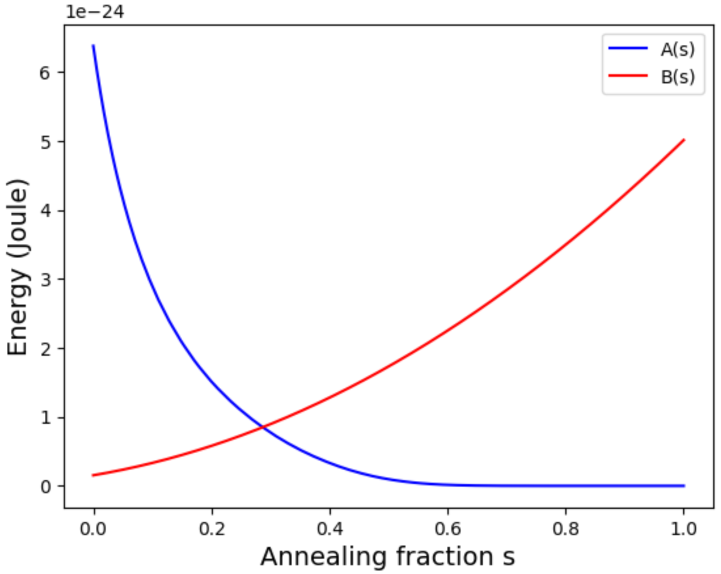
- D-Wave QA Hamiltonian

$$H(s) = A(s)H_{\mathrm{M}} + B(s)H_{\mathrm{P}}$$

$$H_{\mathrm{M}} = \sum_{v\in V_{\mathrm{s}}} \sigma_v^x$$

$$H_{\mathrm{P}} = \sum_{v\in V_{\mathrm{s}}} h_v\sigma_v^z + \sum_{(u,v)\in E_{\mathrm{s}}} J_{uv}\sigma_u^z\sigma_v^z$$

D-Wave Advantage 6.4 Annealing Schedule



- Eigen energies decomposition of the Hamiltonian as fixed s:

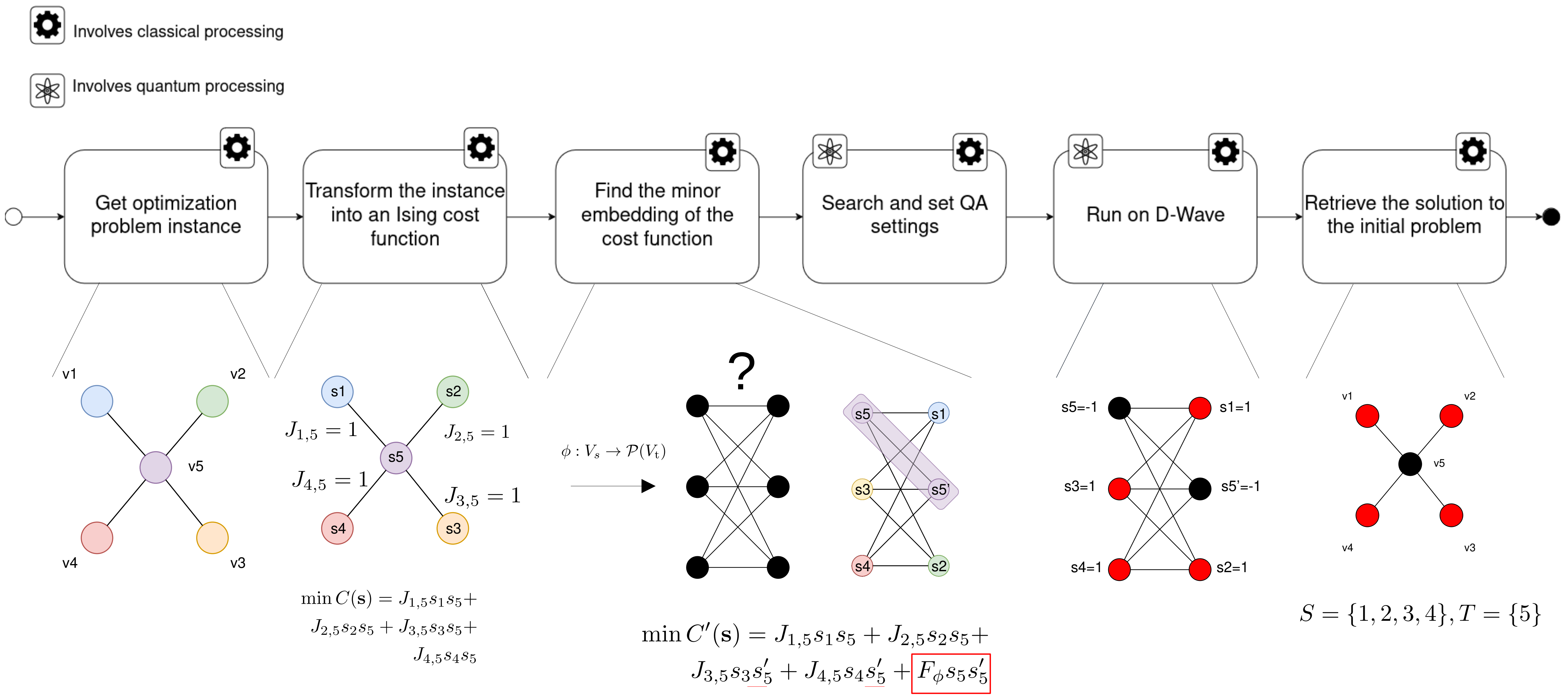
$$H_s\,|\psi_i(s)\rangle = E_i(s)\,|\psi_i(s)\rangle \ \text{ with } E_0(s) < E_1(s) < \ldots < E_k(s)$$

- Minimum spectral gap (the bigger, the better !)

$$\Delta_{\min} = \min_{0\leq s\leq 1}\left(E_1(s) - E_0(s)\right)$$

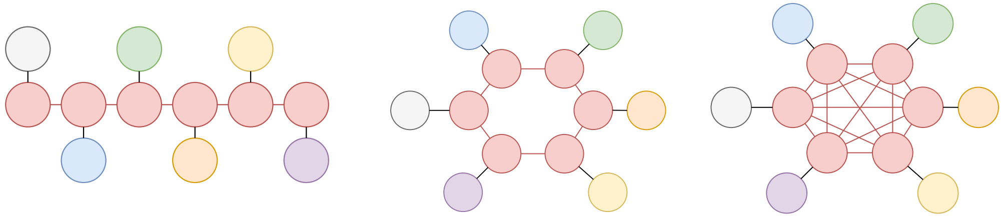
$$T > O(1/\Delta_{\min}^2)$$

I- Context & Motivations – QA programming flow



I- Context & Motivations – Research question

- How does the **topology of physical qubits** associated with a single logical qubit impact the chain strength and the resolution of the problem ?
  - How does the arrangement of the physical qubits impact the spectral gap ?



  - Which arrangement should be priviledged ?

- How to efficiently set the chain strength F_ϕ ?
  - Conservation of the minimum optimal solution

$$\arg\min C = \arg\min C'$$

  - Why is this value hard to set ?
    - Sweet spot usually determined experimentally. Strong but not too strong.
    - Imply auto-coupler and coupler strength rescaling.
    - Changes the problem Hamiltonian and can induce Integrated Control Errors (ICE) [1]

II- State of the Art

- Topology of Physical Qubits
  - Exploration of 4-clique minor-embedding [2]
    - Provide stronger qubits couplings
    - Reduce the chain break

- Chain strength F_ϕ
  - Upper-bound on individual qubits [3]:

$$F_{\phi(v)} < -\left(|h_i| + \sum_{j \in \mathrm{nbr}(i)} |J_{ij}|\right) \qquad O(D)$$

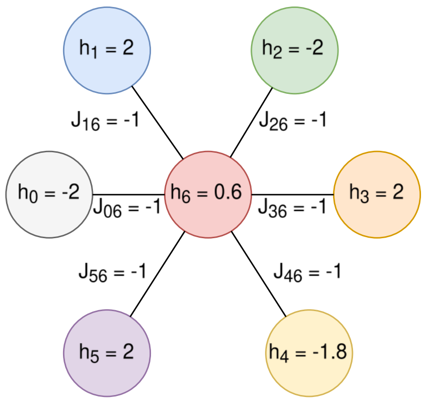
  - Other bounds:
    - Tighter bound in $O(DL)$ [3]
    - Exact Bound in $O(D2^L)$ [4]
  - Average scaling of spin glass problems (e.g. SK problem [5] and default D-Wave method [1])

$$F_\phi \sim \tau\sqrt{n}$$

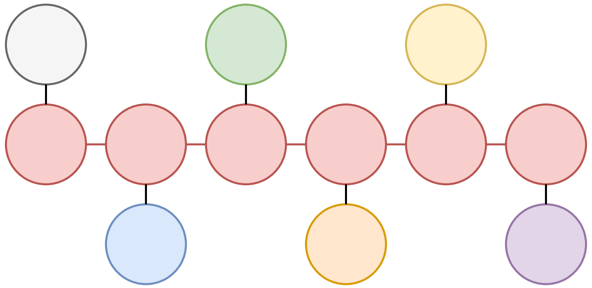
  - Chain scan method [6] [7]
    - Use of the expectation value to optimize chain strength
  - Other optimization approaches (Lagrangian [8] and Genetic-based [9] approaches)

III- Shape of the logical qubit

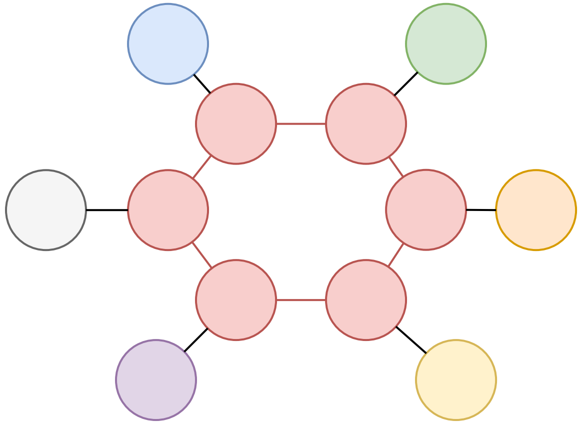
Source graph



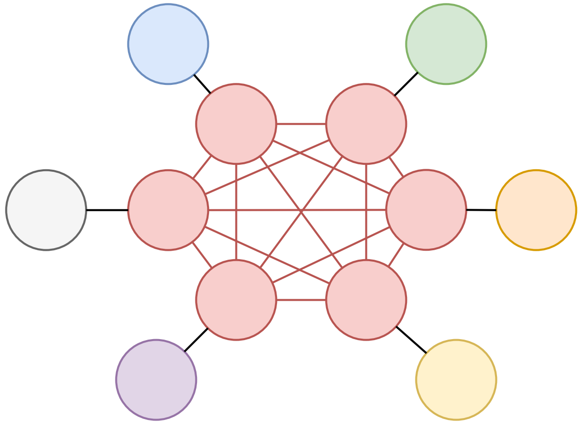
Chain encoding

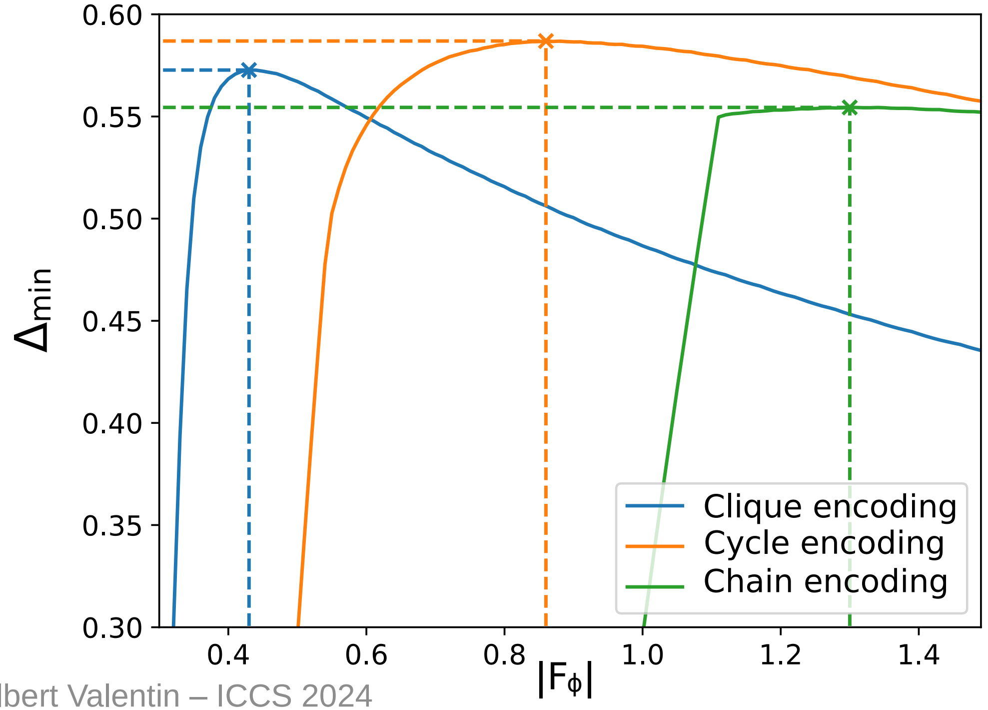


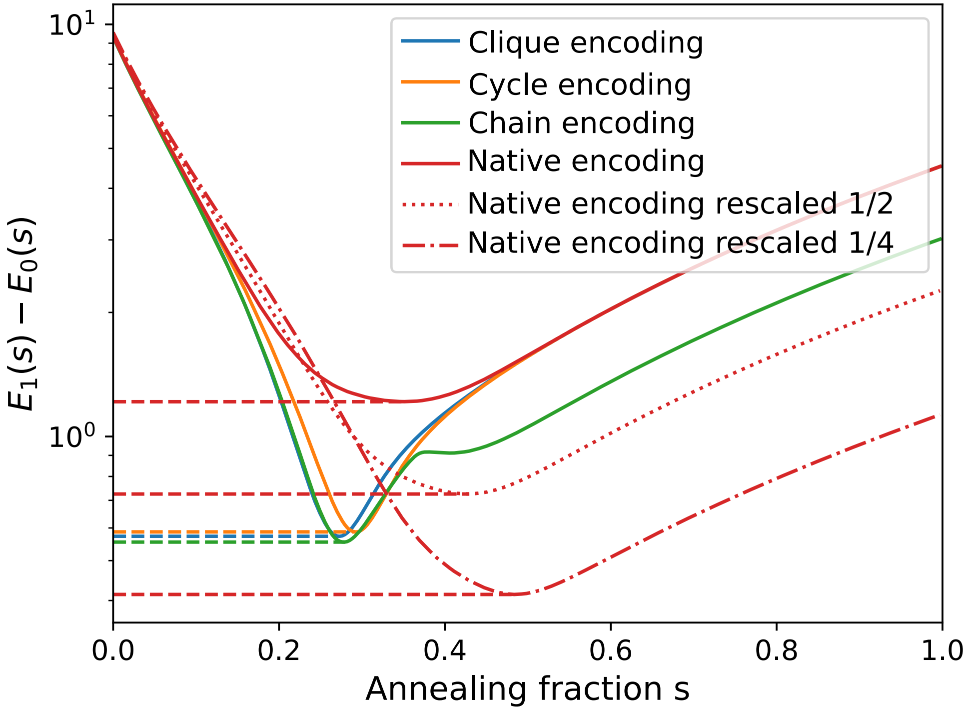
Cycle encoding



Clique encoding





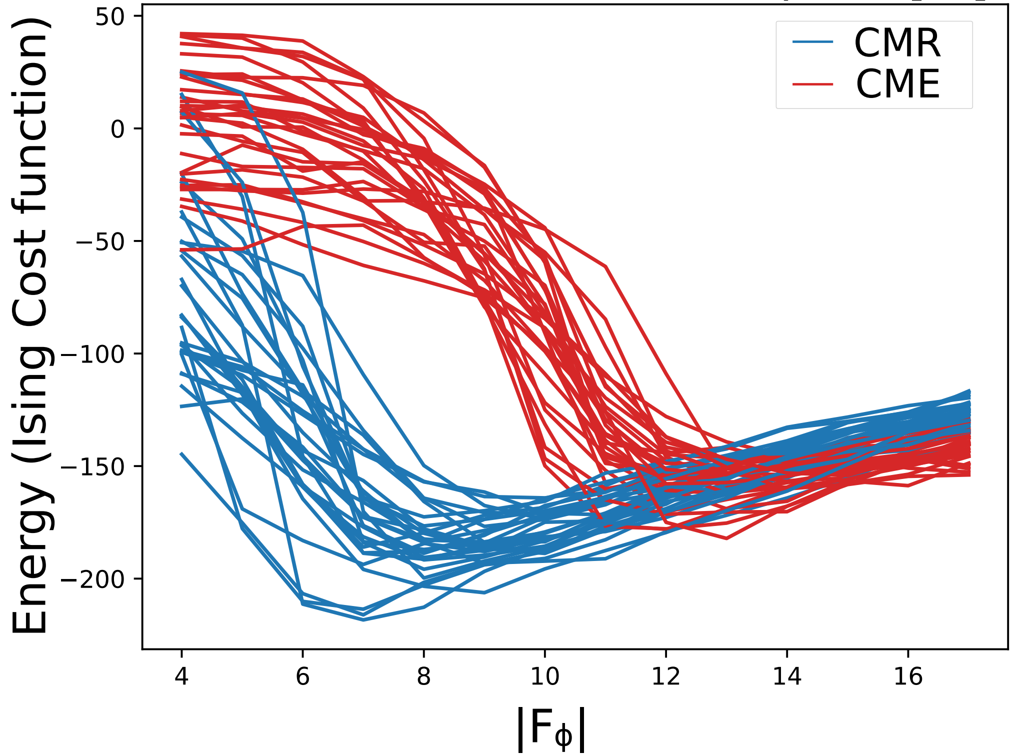


III- Shape of the logical qubit

- How does the arrangement of physical qubits representing a single logical qubit impact the chain strength and the resolution of the problem ?
  - Denser structures seem to require less coupling strength to maintain ferromagnetic coupling and maximize the minimum spectral gap
  - Spectral gap difference is negligible compared to the effect of weight rescaling

  => Different embeddings with an equal number of qubits may have different optimal chain strengths.

- Chain scan for instances embedded with two different methods (CME [10] & CMR [11]):



IV- Setting the chain strength

- Measure of the chain break error rate.

$p_\mathrm{b}(i)$: probability of having a chain break on qubit i

n_lq : number of logical qubits

n_s : number of shots

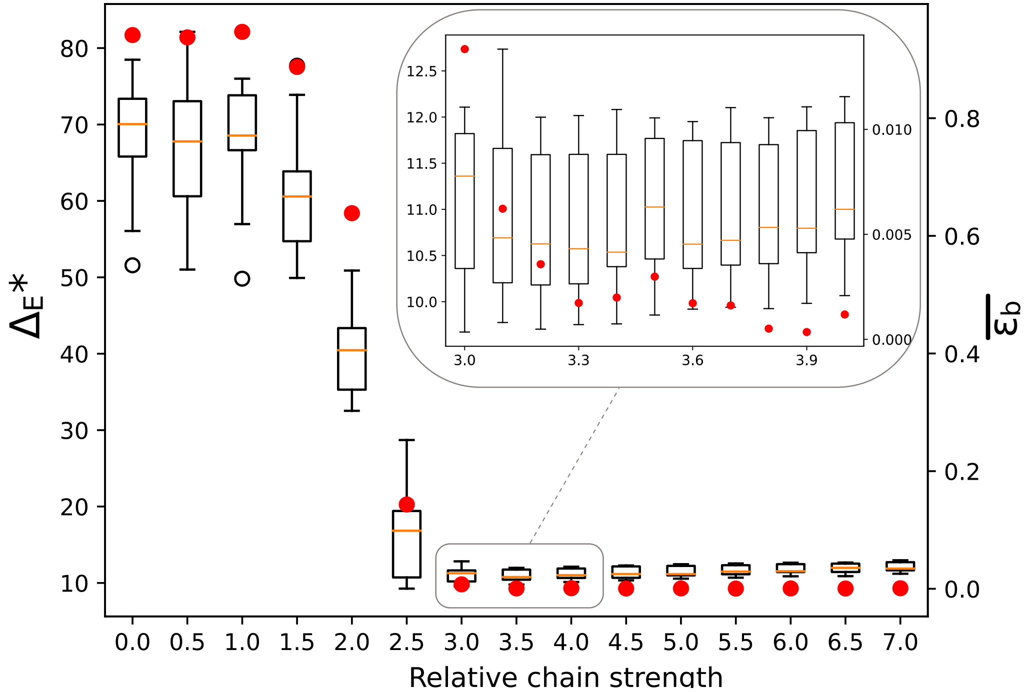
Single shot chain break error rate

$$\epsilon_\mathrm{b} = \frac{1}{n_\mathrm{lq}} \sum_{i=1}^{n_\mathrm{lq}} p_\mathrm{b}(i)$$

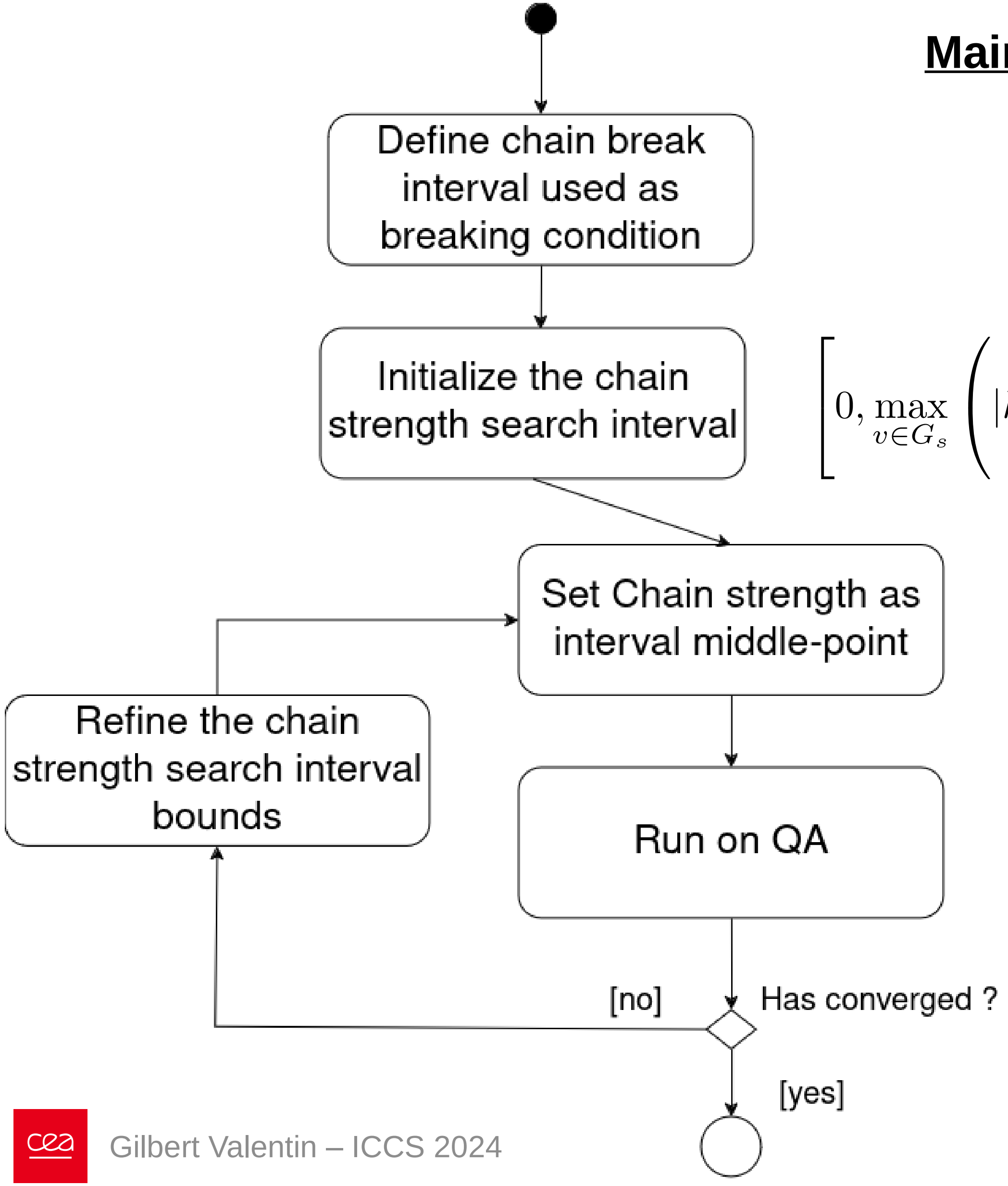
Average chain break error rate

$$\bar{\epsilon_\mathrm{b}} = \frac{1}{n_\mathrm{s}} \sum_{i=1}^{n_\mathrm{s}} \epsilon_\mathrm{b}^{(i)}$$

- Correlation between the chains strength and the duplication error rate (Previous study in [12])



IV- Setting the chain strength – Binary search



$$\left[0, \max_{v \in G_s}\left(|h_i| + \sum_{j \in \mathrm{nbr}(i)} |J_{ij}|\right)\right]$$

Main idea : Use the chain break statistics to drive the search

- Pros of the method:
  - Universal method (can be used for any Ising cost function)
  - Fast convergence
  - Require only a few shots per run
- Cons of the method:
  - Require the setting of a precise chain break interval for the breaking condition
    - May depend on the QC
    - Mays depend on the size of the problem

IV- Setting the chain strength – Results

- 30 instances of unweighted max-cut problem for each density
- Chain break intervals:

  Advantage2:

  $[6 \times 10^{-3}, 2 \times 10^{-2}]$

  Advantage6.4:

  $[2 \times 10^{-2}, 5 \times 10^{-2}]$
- Shot / pre-processing step:

  128
- Final run number of shots:

  Advantage2: 3072

  Advantage6.4: 4096

Advantage2_prototype2.2			Best cut size	Cut size improvement				Step
Instance size	Density	Embedding		min	max	mean	std	
$n = 40$	0.1	CMR	66.4	+0%	+0%	+0%	0%	5.4
	0.5	CMR	243	+0%	+2%	+0.2%	0%	5.8
	0.9	CMR	362.8	+2.1%	+8.2%	+5%	1.6%	4.6
$n = 80$	0.1	CMR	235.5	+0%	+0%	+0%	0%	4.7
	0.5	CME	804	+9.8%	+17.2%	+12.5%	0.2%	4.2
	0.9	CME	1435	+2%	+4.7%	+3.2%	0.6%	4.2
Advantage6.4			Best Cut size	Cut size improvement				Step
Instance size	Density	Embedding		min	max	mean	std	
$n = 100$	0.1	CMR	355.9	+0%	+0.3%	+0%	0%	4.5
	0.5	CME	1271.4	+5.6%	+14.5%	+8.8%	1.8%	2.7
	0.9	CME	2243	+1.4%	+3.7%	+2.5%	0.5%	3.7
$n = 170$	0.1	CMR	950.8	−2.1%	+0.6%	−0.5%	0.5%	2.1
	0.5	CME	3631.4	+2.8%	+6.2%	+4.5%	0.7%	2.1
	0.9	CME	6519.4	+0.4%	+1.4%	+0.8%	0.2%	3.2

V- Discussion & Perspectives

- Conclusion
  - **Optimal chain strength** seems to depend on the **shape of physical qubits** representing the logical qubit(denser mappings require lower chain strength).
  - The chain strength can be optimized only using **duplication error rates**.
  - Our method further optimizes the chain strength value with **few pre-processing shots** (depends on the optimal chain break interval).

- Perspectives
  - Study spectral gap tradeoffs between weight rescaling and adding new qubits to avoid the rescale.
  - Identify and optimize the optimal chain break interval used by our method as the breaking condition.
  - Try this method on other optimization problems (find problems for which the default D-Wave method is not efficient).
  - See if the chain strength can be individually optimized

    => We made some attempts that are not conclusive

Thank You !!

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Outline

- Context & Motivation
- State of the Art
- Topology of physical qubits for a single logical qubit
- Setting the chain strength
- Results