Performance evaluation and control improvements for solving optimization problems on Noisy Intermediate-Scale Quantum (NISQ) platforms

PhD Thesis defense

Valentin GILBERT

18th December 2024

Advisors: Stéphane LOUISE, Renaud SIRDEY

Reviewers: Frank PHILLIPSON, Caroline PRODHON

Examiners: Daniel ESTEVE, Jeannette LORENZ

Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand …)

Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand …)

$$S_{\text{interest}} = 0.03$$

Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand ...)

$$S_{\text{interest}} = 0.8$$

Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand …)

$$S_{\text{interest}} = 0.8$$

- Similarity score: color and shape similarity

Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand ...)

$$S_{\text{interest}} = 0.8$$

- Similarity score: color and shape similarity

$$S_{\text{similarity}} = 0.95$$

Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand ...)

$$S_{\text{interest}} = 0.8$$

- Similarity score: color and shape similarity

$$S_{\text{similarity}} = 0.2$$

Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand ...)

$$S_{\text{interest}} = 0.8$$

- Similarity score: color and shape similarity

$$S_{\text{similarity}} = 0.2$$

Collection representation

Introduction and Context

Collection criteria

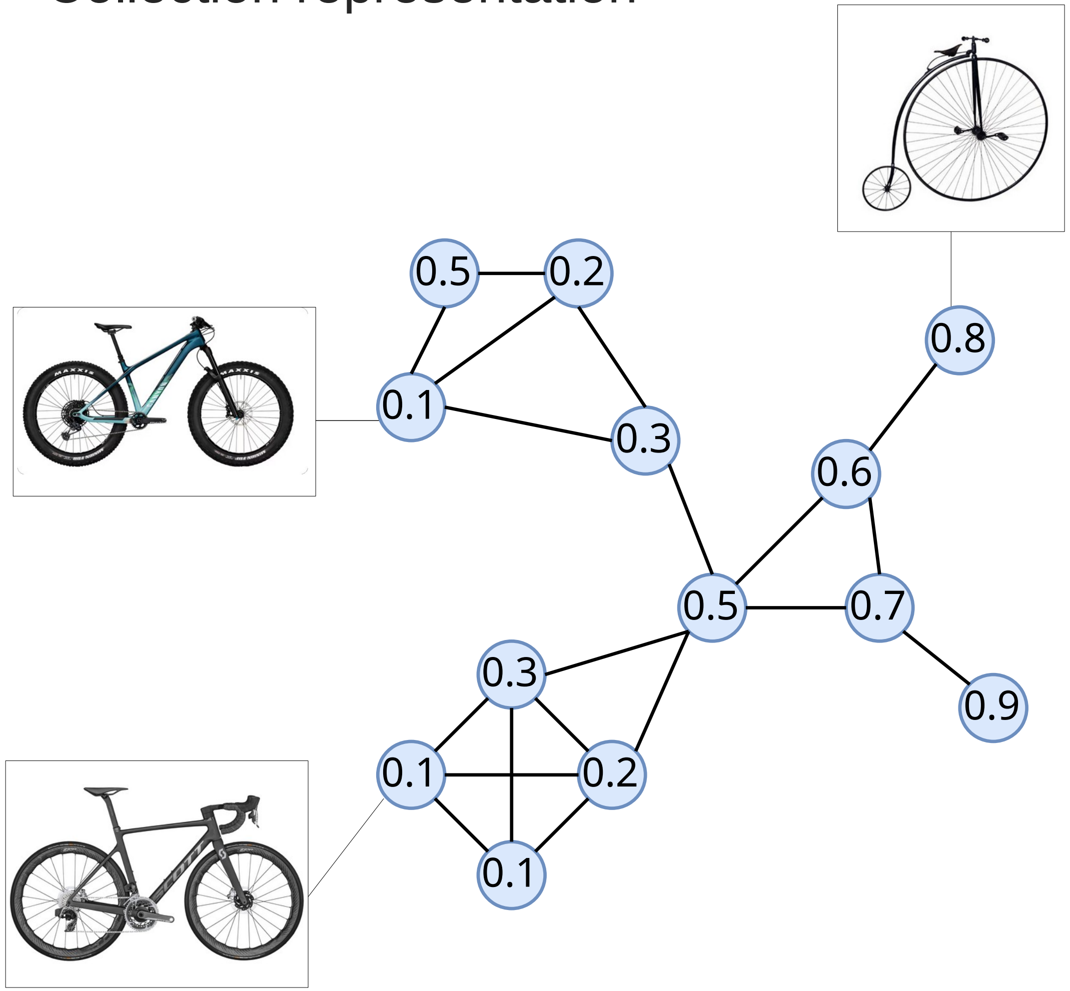
- Score of interest (beauty, color, shape, brand …)

$$S_{\text{interest}} = 0.8$$

- Similarity score: color and shape similarity

$$S_{\text{similarity}} = 0.2$$

Collection representation



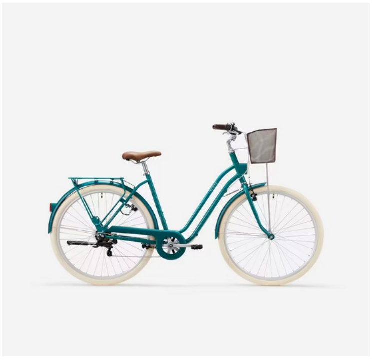
Introduction and Context

Collection criteria

- Score of interest (beauty, color, shape, brand ...)

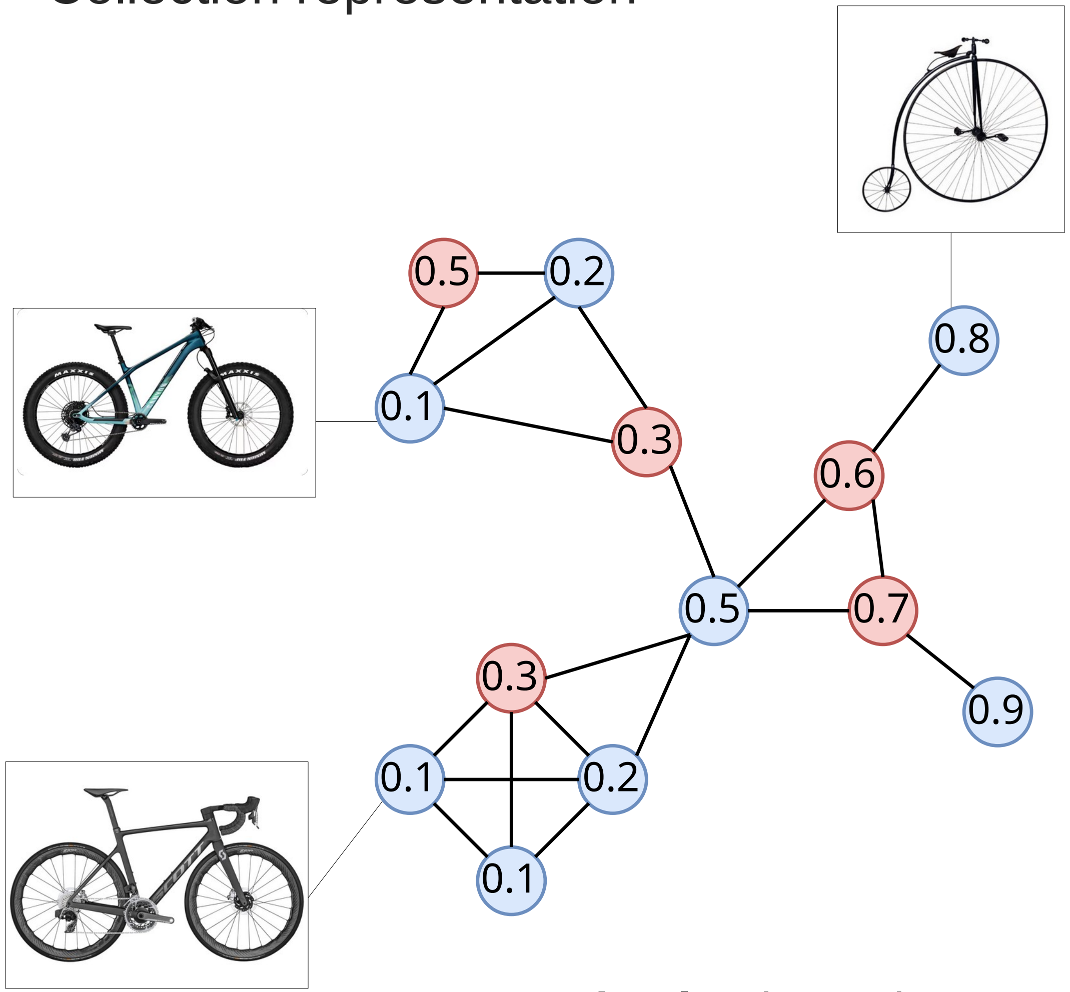
$$S_{\text{interest}} = 0.8$$

- Similarity score: color and shape similarity



$$S_{\text{similarity}} = 0.2$$

Collection representation



Maximal Independent Set

Introduction and Context

Collection criteria

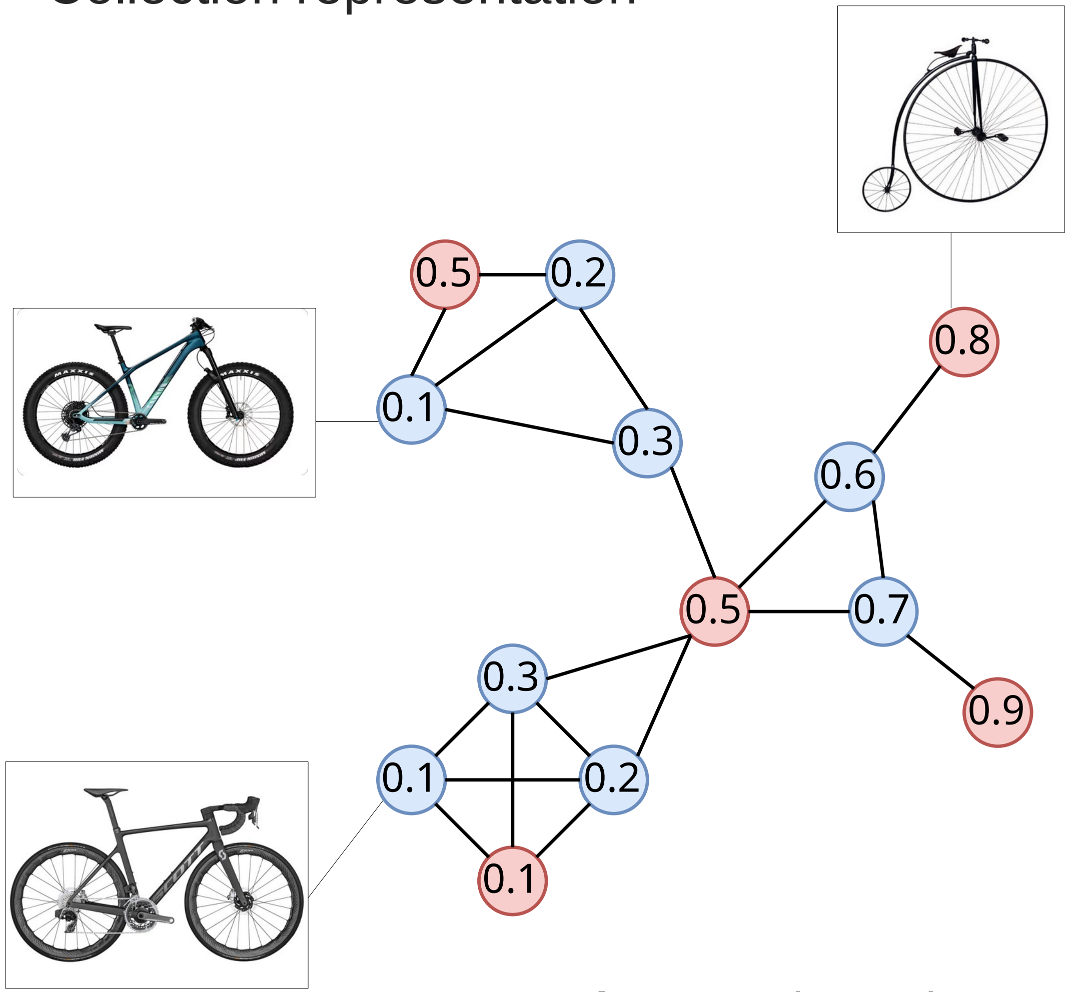
- Score of interest (beauty, color, shape, brand ...)

$$S_{\text{interest}} = 0.8$$

- Similarity score: color and shape similarity

$$S_{\text{similarity}} = 0.2$$

Collection representation



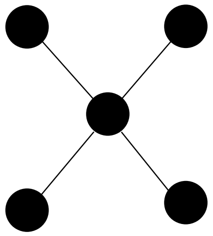
Maximum Independent Set

Introduction and Context

Maximum Independent Set (MIS) problem

$$G = (V_s, E_s)$$

$$x_v \in \{0, 1\}$$

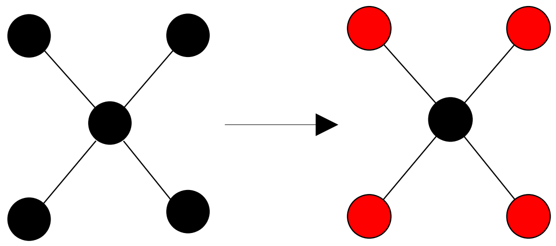


Introduction and Context

Maximum Independent Set (MIS) problem

$$G = (V_s, E_s)$$

$$x_v \in \{0, 1\}$$

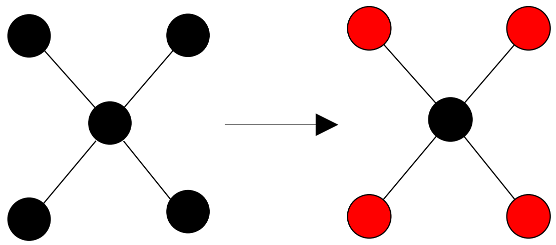


Introduction and Context

Maximum Independent Set (MIS) problem

$$G = (V_s, E_s)$$

$$x_v \in \{0, 1\}$$



- Problem formulation

$$\max \sum_{v \in V_s} x_v$$

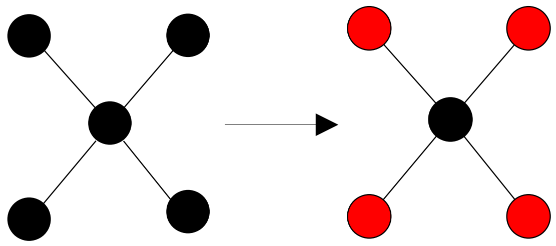
$$\text{subject to } x_u \neq x_v \text{ if } (u, v) \in E_s$$

Introduction and Context

Maximum Independent Set (MIS) problem

$$G = (V_s, E_s)$$

$$x_v \in \{0, 1\}$$



- Problem formulation

$$\max \sum_{v \in V_s} x_v$$

$$\text{subject to } x_u \neq x_v \text{ if } (u, v) \in E_s$$

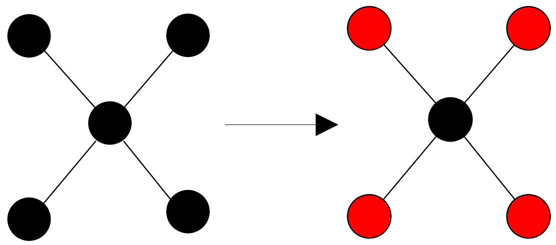
$$\max \sum_{v \in V_s} x_v - 2 \sum_{(u,v) \in E_s} x_u x_v$$

Introduction and Context

Maximum Independent Set (MIS) problem

$$G = (V_s, E_s)$$

$$x_v \in \{0, 1\}$$



- Problem formulation

$$\max \sum_{v \in V_s} x_v$$

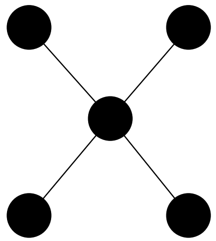
$$\text{subject to } x_u \neq x_v \text{ if } (u, v) \in E_s$$

$$\max \sum_{v \in V_s} x_v - 2 \sum_{(u,v) \in E_s} x_u x_v$$

Max-cut problem

$$G = (V_s, E_s)$$

$$s_v \in \{-1, +1\}$$

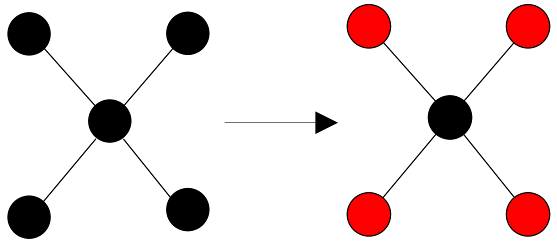


Introduction and Context

Maximum Independent Set (MIS) problem

$$G = (V_s, E_s)$$

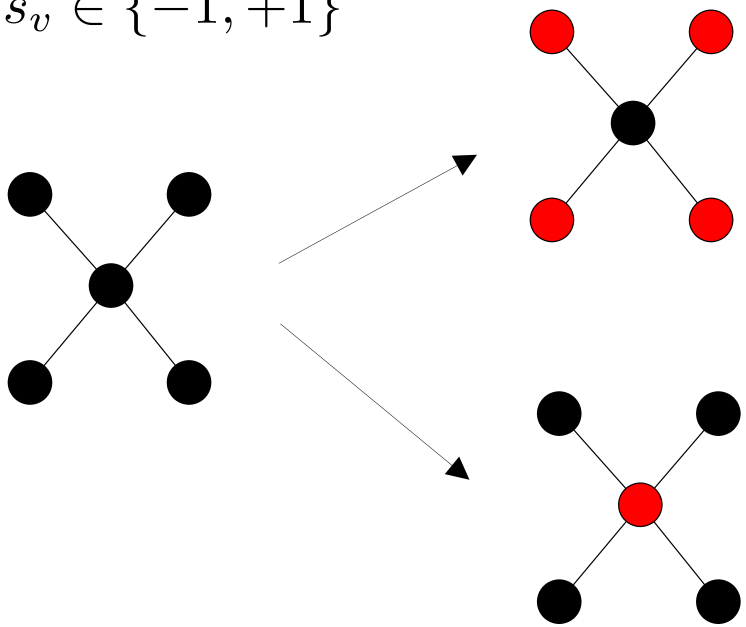
$$x_v \in \{0, 1\}$$



Max-cut problem

$$G = (V_s, E_s)$$

$$s_v \in \{-1, +1\}$$



- Problem formulation

$$\max \sum_{v \in V_s} x_v$$

$$\text{subject to } x_u \neq x_v \text{ if } (u, v) \in E_s$$

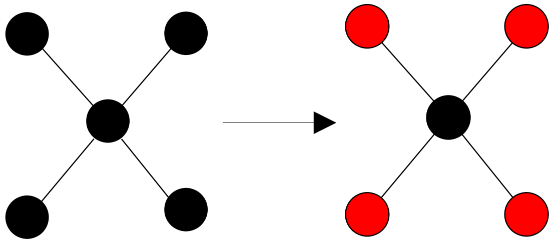
$$\max \sum_{v \in V_s} x_v - 2 \sum_{(u,v) \in E_s} x_u x_v$$

Introduction and Context

Maximum Independent Set (MIS) problem

$$G = (V_s, E_s)$$

$$x_v \in \{0, 1\}$$



- Problem formulation

$$\max \sum_{v \in V_s} x_v$$

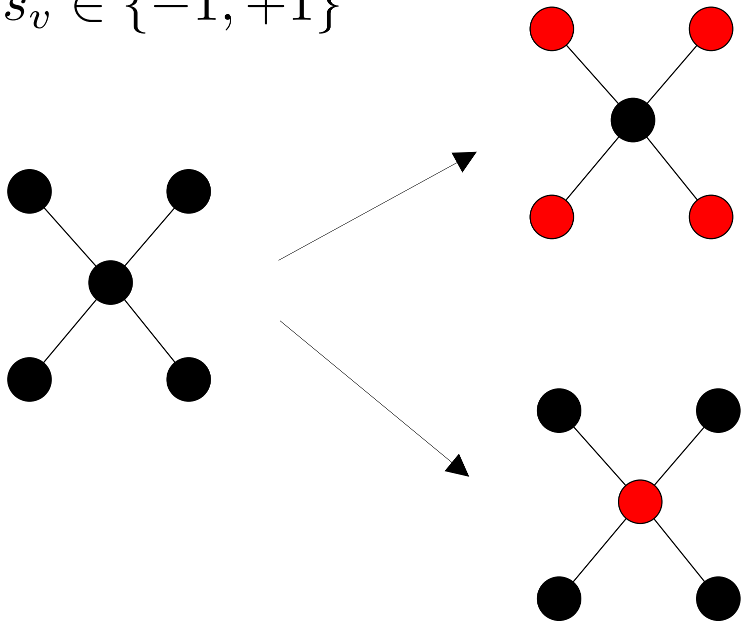
$$\text{subject to } x_u \neq x_v \text{ if } (u, v) \in E_s$$

$$\max \sum_{v \in V_s} x_v - 2 \sum_{(u,v) \in E_s} x_u x_v$$

Max-cut problem

$$G = (V_s, E_s)$$

$$s_v \in \{-1, +1\}$$



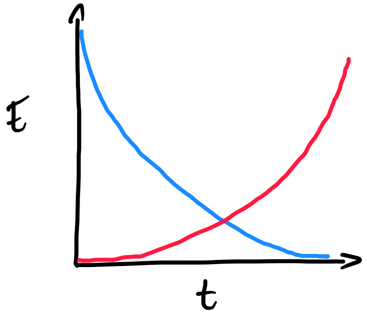
- Problem formulation

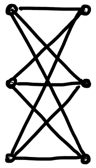
$$\max - \sum_{(u,v) \in E_s} s_u s_v$$

Introduction - Quantum approaches for optimization problems

Introduction - Quantum approaches for optimization problems

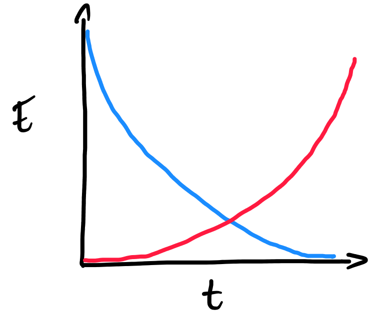
- Quantum Annealers (analog-based) (NISQ) [KN98]

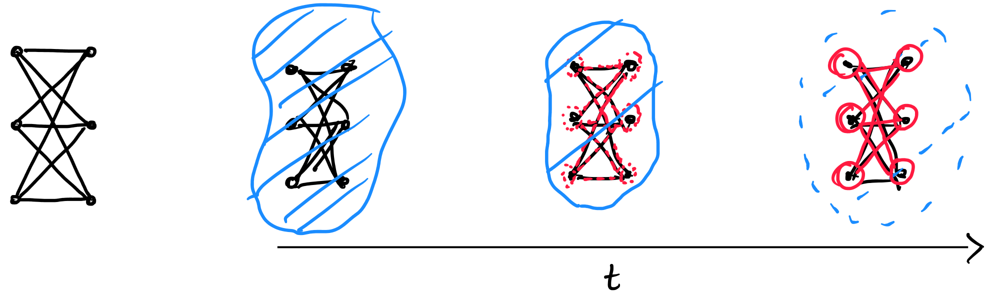




Introduction - Quantum approaches for optimization problems

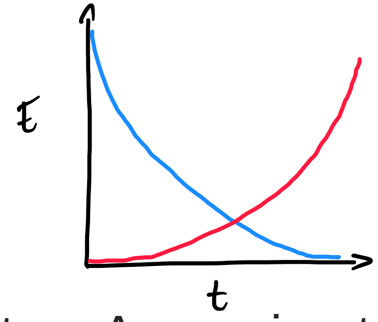
- Quantum Annealers (analog-based) (NISQ) [KN98]

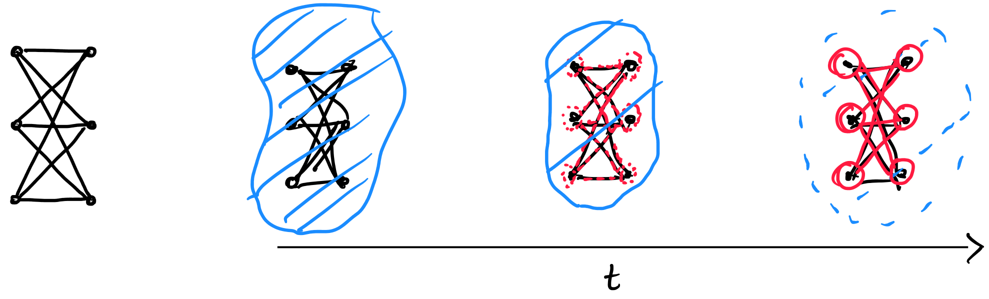




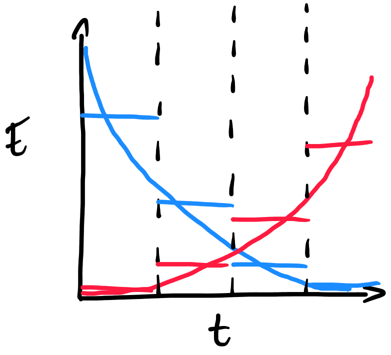
Introduction - Quantum approaches for optimization problems

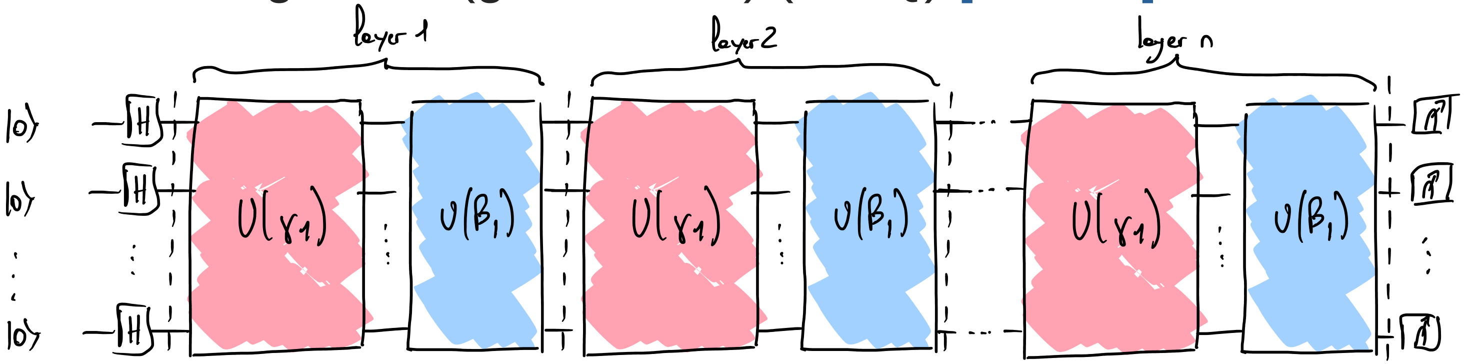
- Quantum Annealers (analog-based) (NISQ) [KN98]





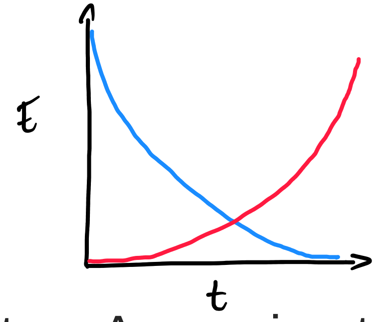
- The Quantum Approximate Optimization Algorithm (gate-based) (NISQ) [FGS14]

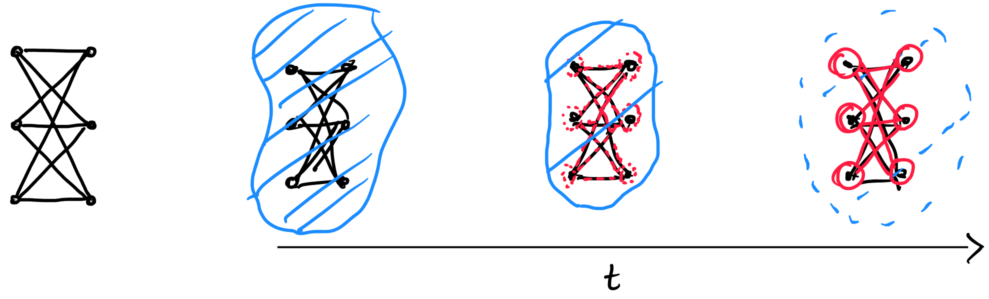




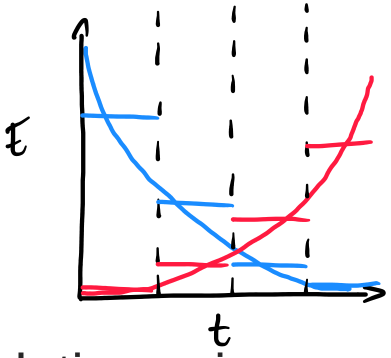
Introduction - Quantum approaches for optimization problems

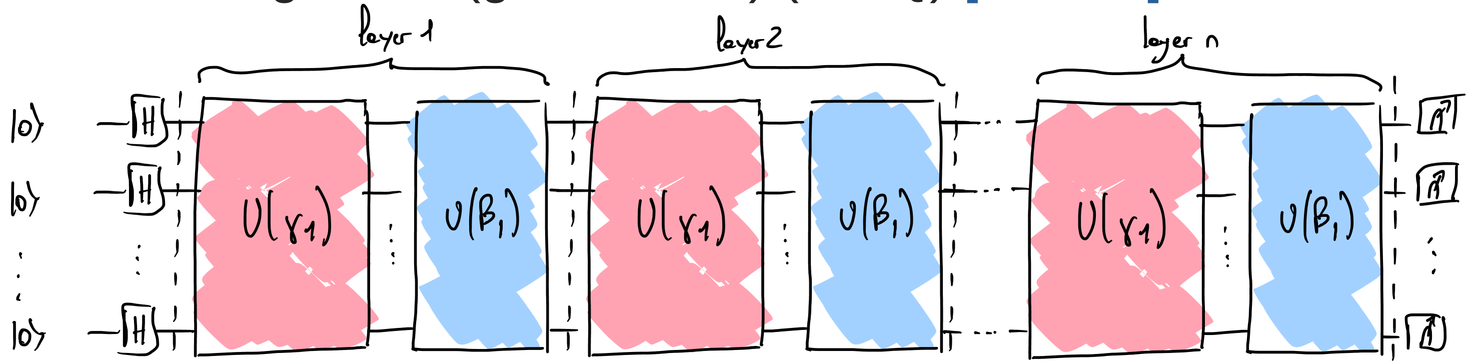
- Quantum Annealers (analog-based) (NISQ) [KN98]



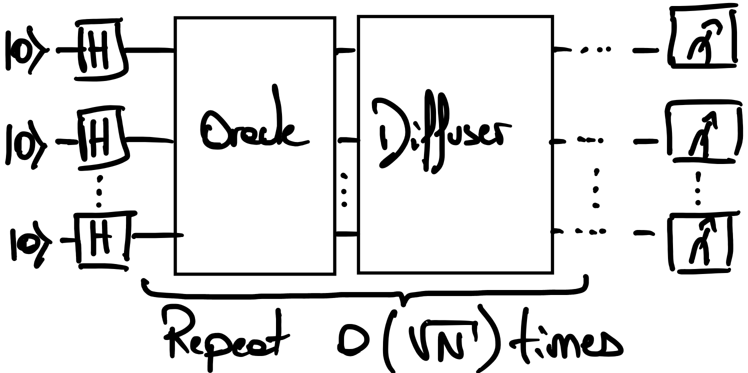


- The Quantum Approximate Optimization Algorithm (gate-based) (NISQ) [FGS14]



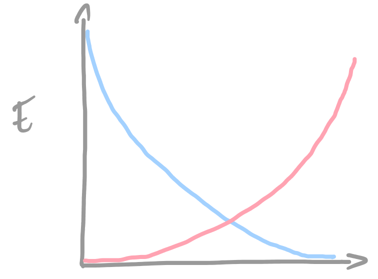


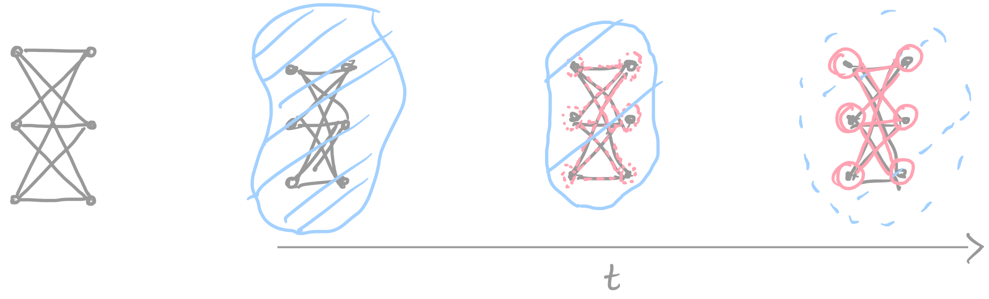
- Exact resolution using quantum circuits (gate-based) (not NISQ) [Gro96]



Introduction - Quantum approaches for optimization problems

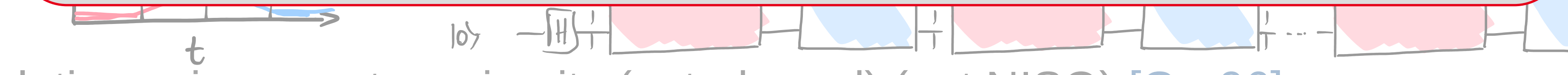
- Quantum Annealers (analog-based) (NISQ) [KN98]

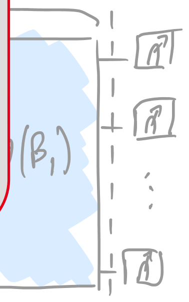




- The Quantu … tion Algorithm (gate-based) (NISQ) [FGS14]

> **Research question**
>
> - What makes a Quantum Annealer perform well?
> - How to improve their performance?

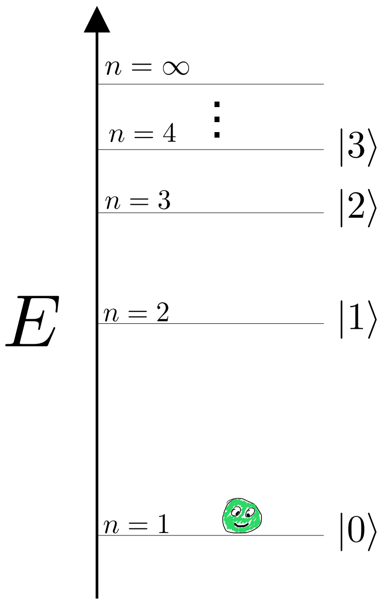




- Exact resolution using quantum circuits (gate-based) (not NISQ) [Gro96]

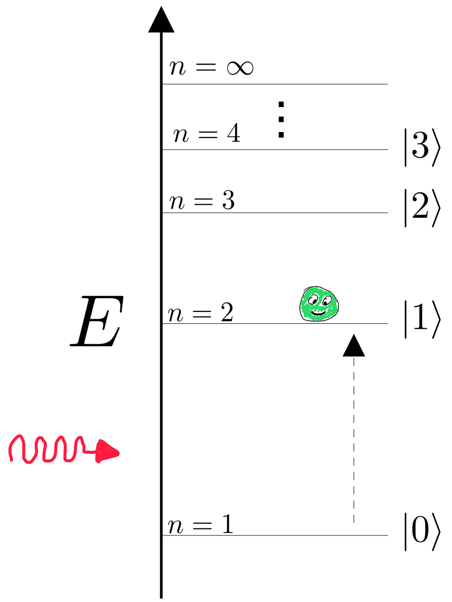
1. Introduction to Quantum Annealing

1- Ground state



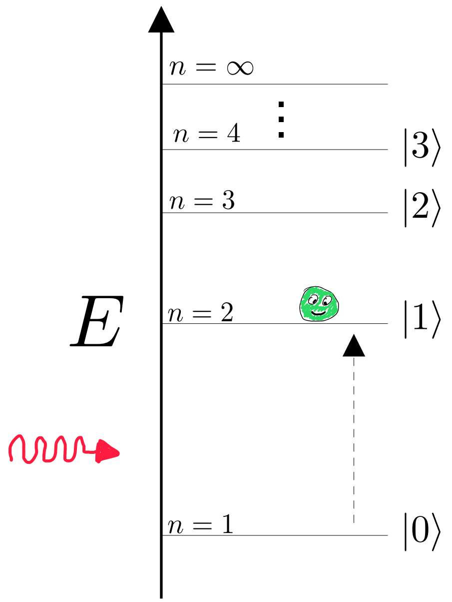
1- Ground state

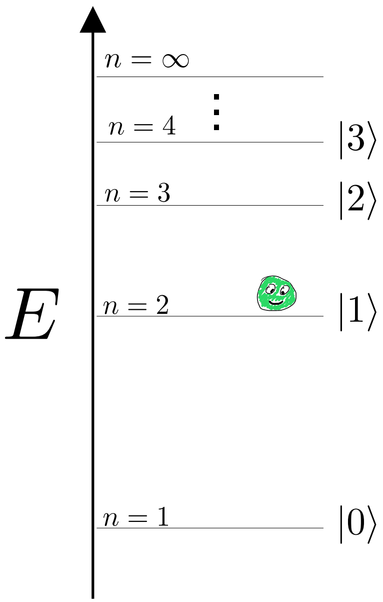
Photon absorption



1- Ground state

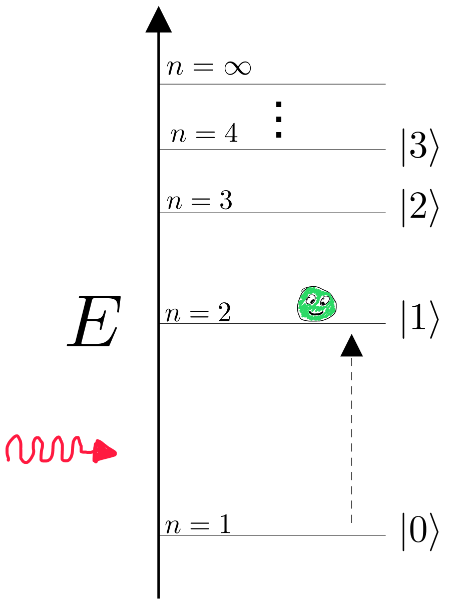
Photon absorption



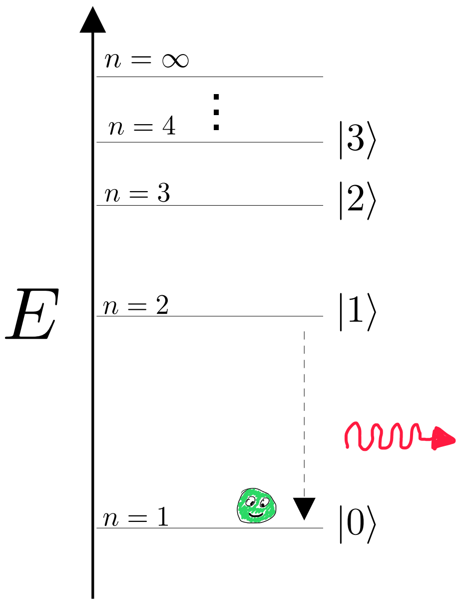


1- Ground state

Photon absorption

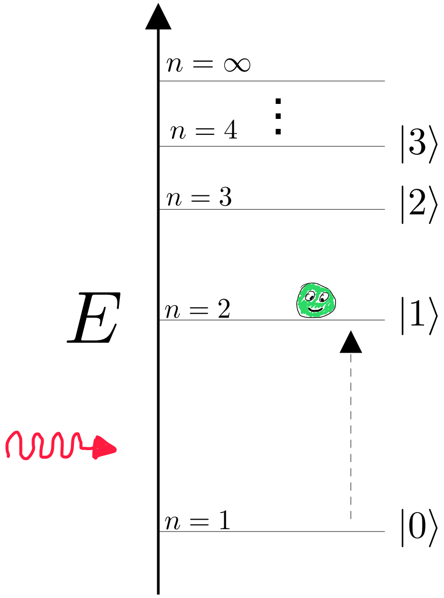


Photon emission

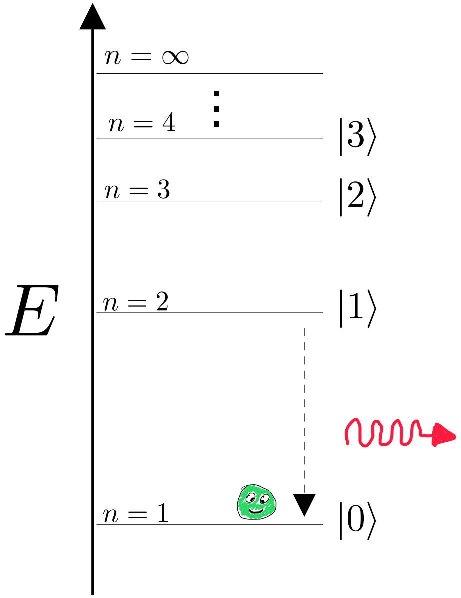


1- Ground state

Photon absorption



Photon emission

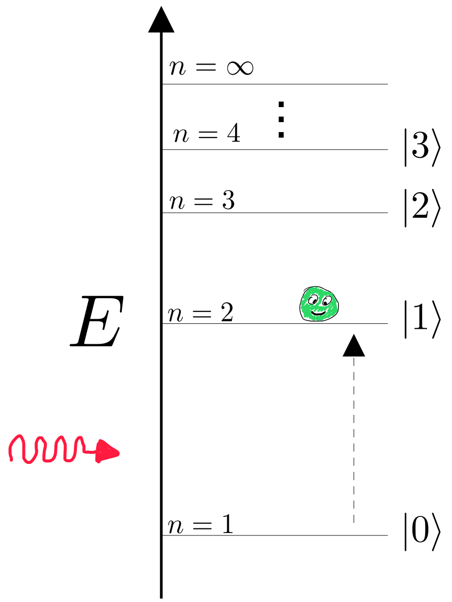


1- Ground state

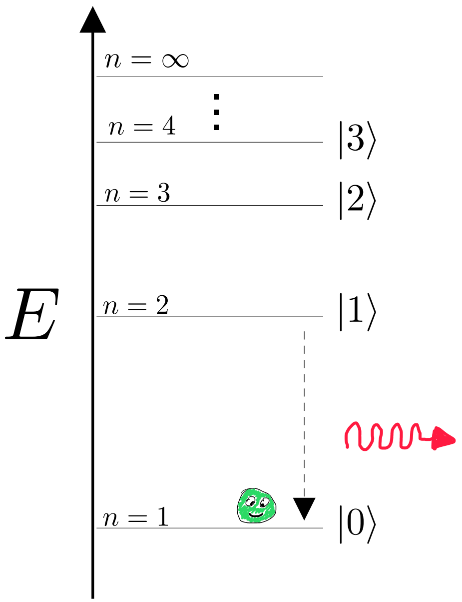
- Hamiltonian

  Energy of distinguishable states

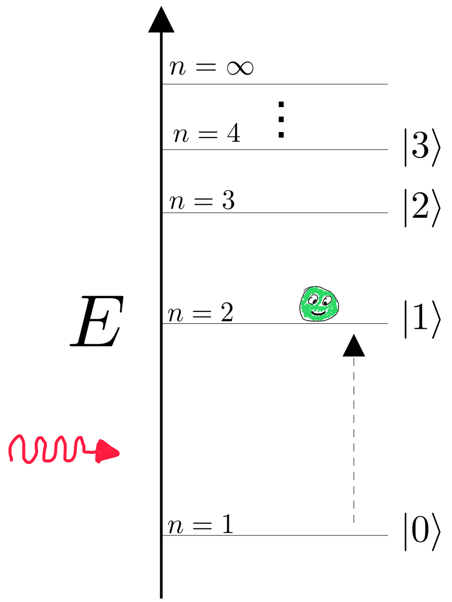
Photon absorption

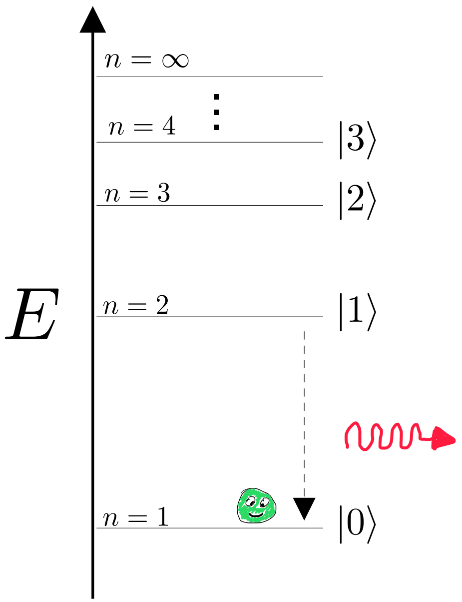


Photon emission



1- Ground state



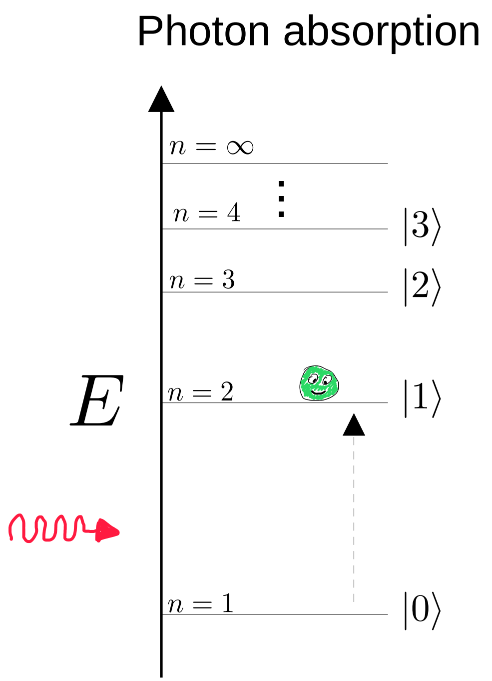


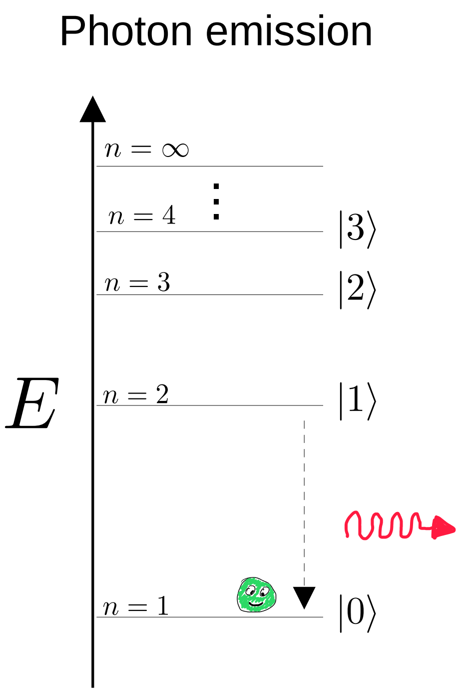
- Hamiltonian

  Energy of distinguishable states

  $$H\,|0\rangle = E_0\,|0\rangle \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

1- Ground state





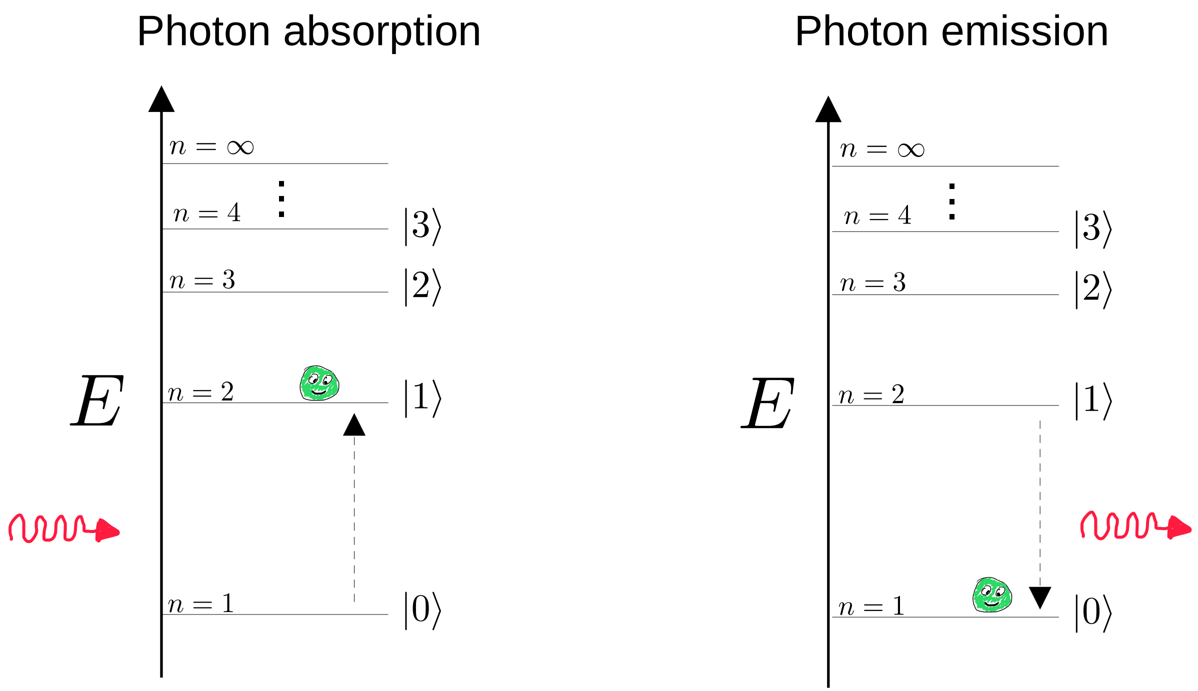
- Hamiltonian

  Energy of distinguishable states

$$H\,|0\rangle = E_0\,|0\rangle \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$H\,|1\rangle = E_1\,|1\rangle \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

1- Ground state



- Hamiltonian

  Energy of distinguishable states

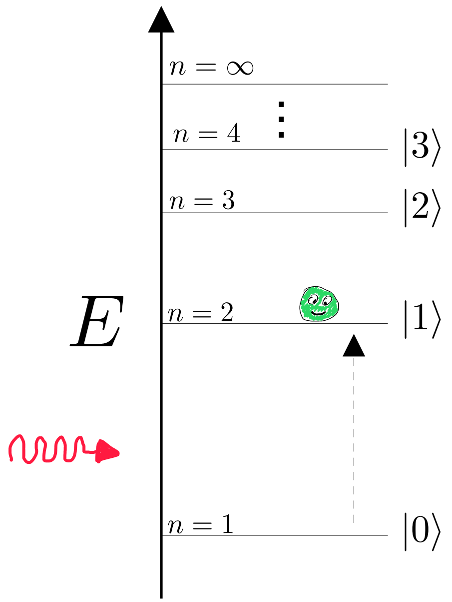
$$H|0\rangle = E_0|0\rangle \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$H|1\rangle = E_1|1\rangle \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

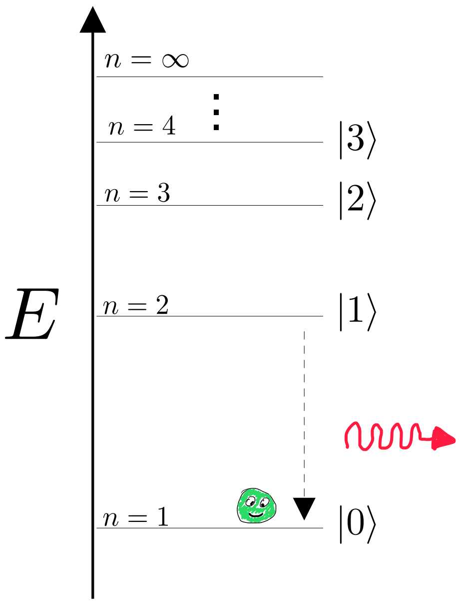
- n-qubit Hamiltonian

1- Ground state

Photon absorption



Photon emission



- Hamiltonian

  Energy of distinguishable states

$$H|0\rangle = E_0|0\rangle \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

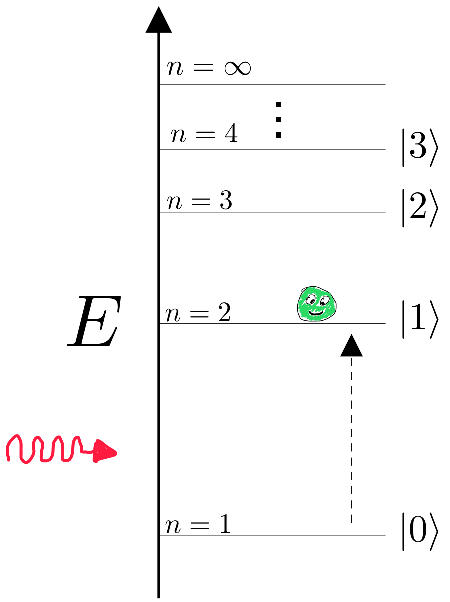
$$H|1\rangle = E_1|1\rangle \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

- n-qubit Hamiltonian

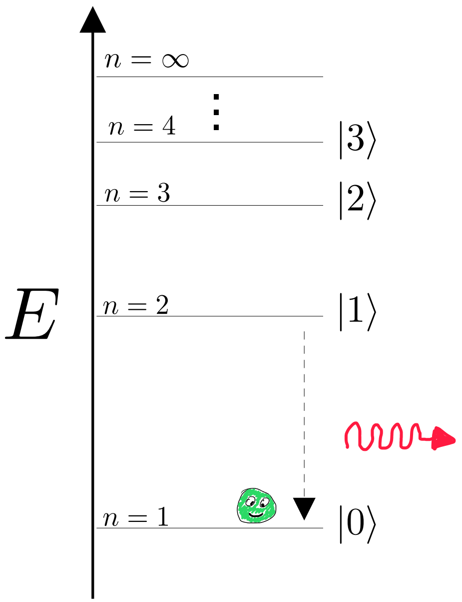
$$H|\psi_p\rangle = E_p|\psi_p\rangle$$

1- Ground state

Photon absorption



Photon emission



- Hamiltonian

  Energy of distinguishable states

  $$H|0\rangle = E_0|0\rangle \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

  $$H|1\rangle = E_1|1\rangle \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

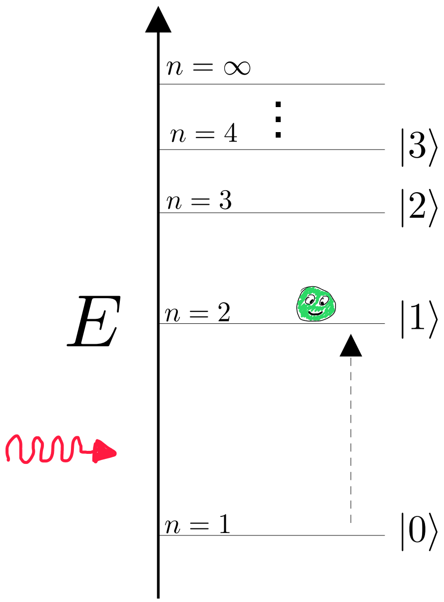
- n-qubit Hamiltonian

  $$H|\psi_p\rangle = E_p|\psi_p\rangle$$

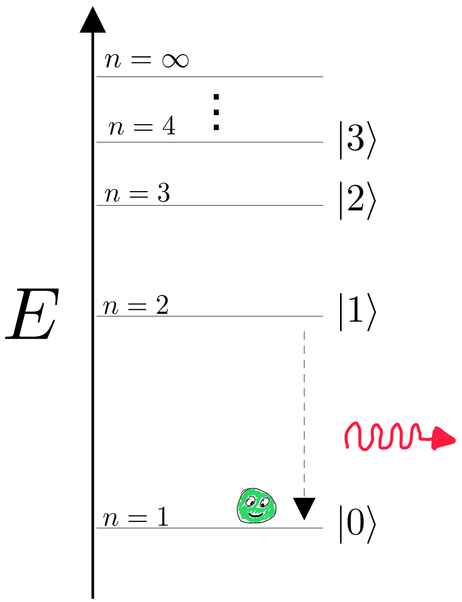
  eigenvalue (E_p) eigenvector ($|\psi_p\rangle$)

1- Ground state

Photon absorption



Photon emission



- Hamiltonian

  Energy of distinguishable states

$$H|0\rangle = E_0|0\rangle \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$H|1\rangle = E_1|1\rangle \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

- n-qubit Hamiltonian

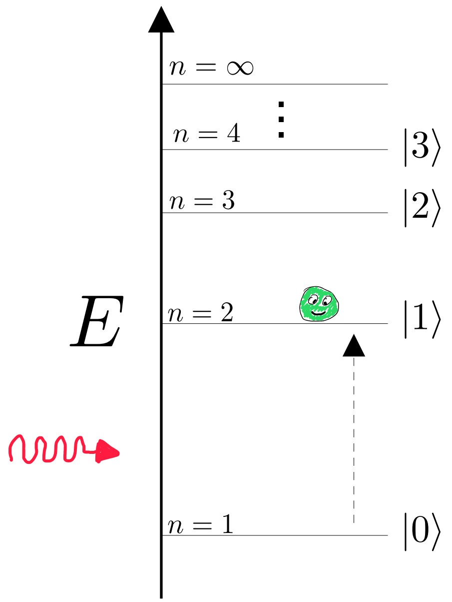
$$H|\psi_p\rangle = E_p|\psi_p\rangle$$

eigenvalue eigenvector

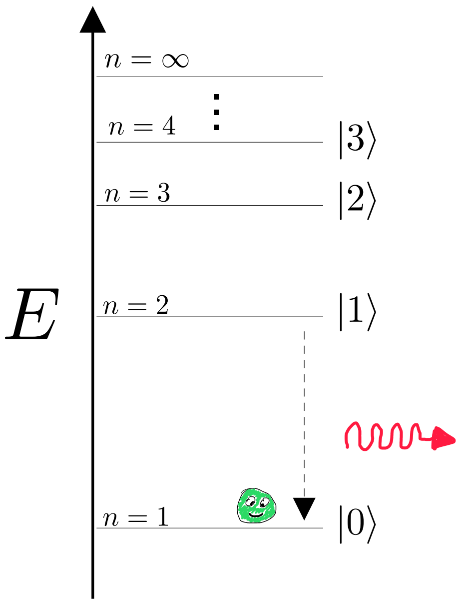
- Time-dependent Hamiltonian

1- Ground state

Photon absorption



Photon emission



- Hamiltonian

  Energy of distinguishable states

$$H|0\rangle = E_0|0\rangle \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$H|1\rangle = E_1|1\rangle \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

- n-qubit Hamiltonian

$$H|\psi_p\rangle = E_p|\psi_p\rangle$$

eigenvalue eigenvector

- Time-dependent Hamiltonian

$$H(t)|\psi_p(t)\rangle = E_p(t)|\psi_p(t)\rangle$$

1- Adiabatic theorem

Quantum Adiabatic Theorem

"A quantum system described by a time-dependent Hamiltonian $H(t)$ initially prepared in an eigenstate $|\psi_p(0)\rangle$ of $H(0)$ (e.g., the ground state), will approximately remain in the instantaneous eigenstate $|\psi_p(t)\rangle$, given that t varies sufficiently slowly"

1- Adiabatic theorem

Quantum Adiabatic Theorem

"A quantum system described by a time-dependent Hamiltonian $H(t)$ initially prepared in an eigenstate $|\psi_p(0)\rangle$ of $H(0)$ (e.g., the ground state), will approximately remain in the instantaneous eigenstate $|\psi_p(t)\rangle$, given that t varies sufficiently slowly"

- Linear interpolation of Hamiltonians:

$$H(t) = -A(t)H_{init} + B(t)H_{final}$$

$$t \in [0, T]$$

1- Adiabatic theorem

Quantum Adiabatic Theorem

"A quantum system described by a time-dependent Hamiltonian $H(t)$ initially prepared in an eigenstate $|\psi_p(0)\rangle$ of $H(0)$ (e.g., the ground state), will approximately remain in the instantaneous eigenstate $|\psi_p(t)\rangle$, given that t varies sufficiently slowly"

- Linear interpolation of Hamiltonians:

$$H(t) = -A(t)H_{init} + B(t)H_{final}$$

$$t \in [0, T]$$

$$A(0) \gg B(0)$$

1- Adiabatic theorem

Quantum Adiabatic Theorem

"A quantum system described by a time-dependent Hamiltonian $H(t)$ initially prepared in an eigenstate $|\psi_p(0)\rangle$ of $H(0)$ (e.g., the ground state), will approximately remain in the instantaneous eigenstate $|\psi_p(t)\rangle$, given that t varies sufficiently slowly"

- Linear interpolation of Hamiltonians:

$$H(t) = -A(t)H_{init} + B(t)H_{final}$$

$$t \in [0, T]$$

$$A(0) \gg B(0)$$

$$A(T) \ll B(T)$$

1- Adiabatic theorem

Quantum Adiabatic Theorem

"A quantum system described by a time-dependent Hamiltonian $H(t)$ initially prepared in an eigenstate $|\psi_p(0)\rangle$ of $H(0)$ (e.g., the ground state), will approximately remain in the instantaneous eigenstate $|\psi_p(t)\rangle$, given that t varies sufficiently slowly"

- Linear interpolation of Hamiltonians:

$$H(t) = -A(t)H_{init} + B(t)H_{final}$$

$$t \in [0, T]$$

$$A(0) \gg B(0)$$

$$A(T) \ll B(T)$$

- Assumption
  - Ideal closed quantum system

1- Adiabatic theorem

Quantum Adiabatic Theorem

"A quantum system described by a time-dependent Hamiltonian $H(t)$ initially prepared in an eigenstate $|\psi_p(0)\rangle$ of $H(0)$ (e.g., the ground state), will approximately remain in the instantaneous eigenstate $|\psi_p(t)\rangle$, given that t varies sufficiently slowly"

- Linear interpolation of Hamiltonians:

$$H(t) = -A(t)H_{init} + B(t)H_{final}$$

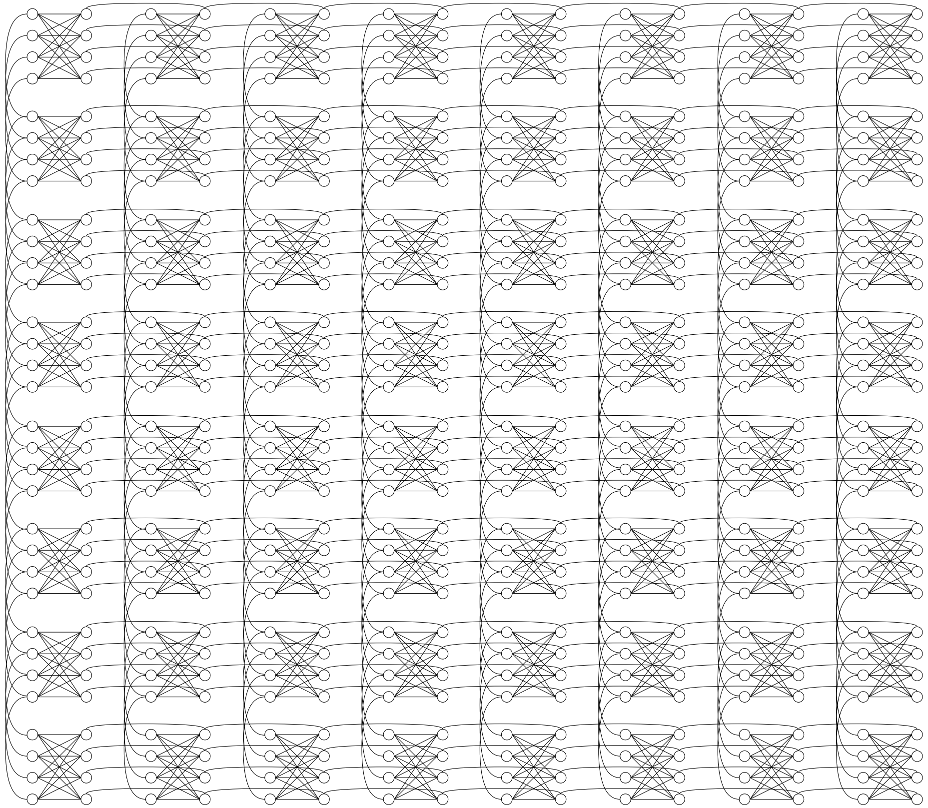
$$t \in [0, T]$$

$$A(0) \gg B(0)$$

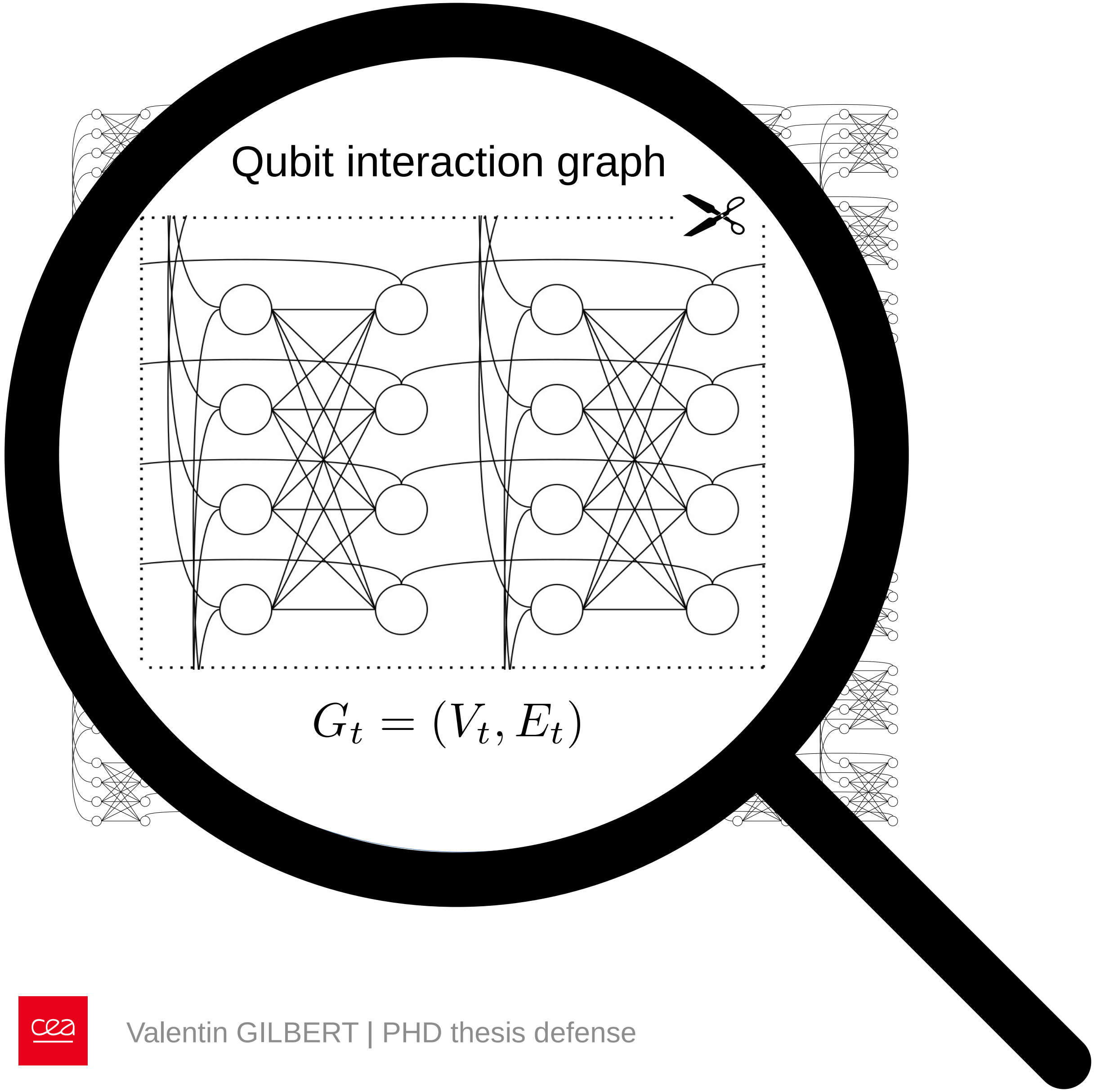
$$A(T) \ll B(T)$$

- Assumption
  - Ideal closed quantum system
- Theoretical advantage:
  - Universal (as quantum circuit)
  - Performance depends on the « sufficiently slowly »

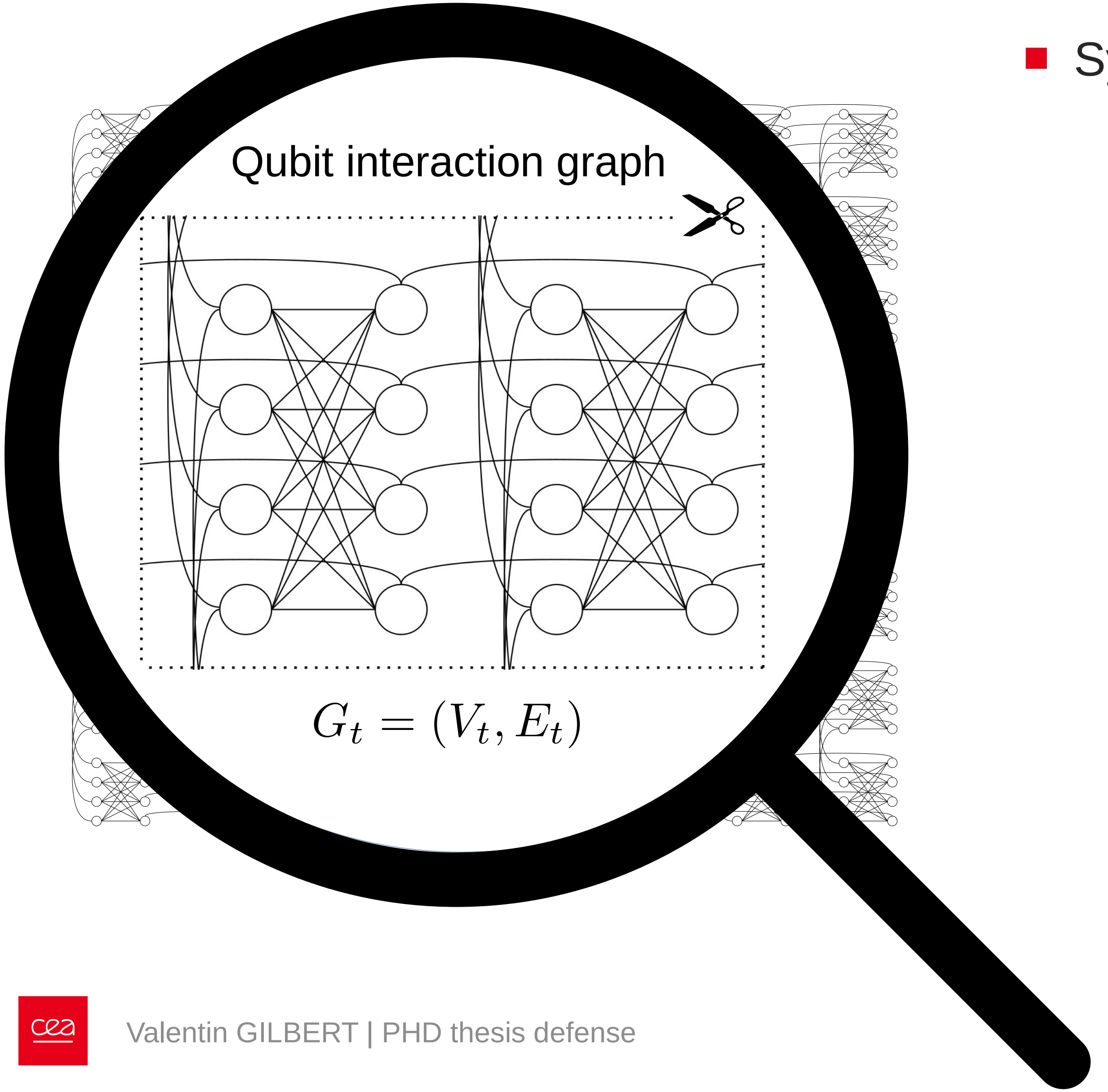
1- Quantum Annealers (D-Wave systems)



1- Quantum Annealers (D-Wave systems)



1- Quantum Annealers (D-Wave systems)

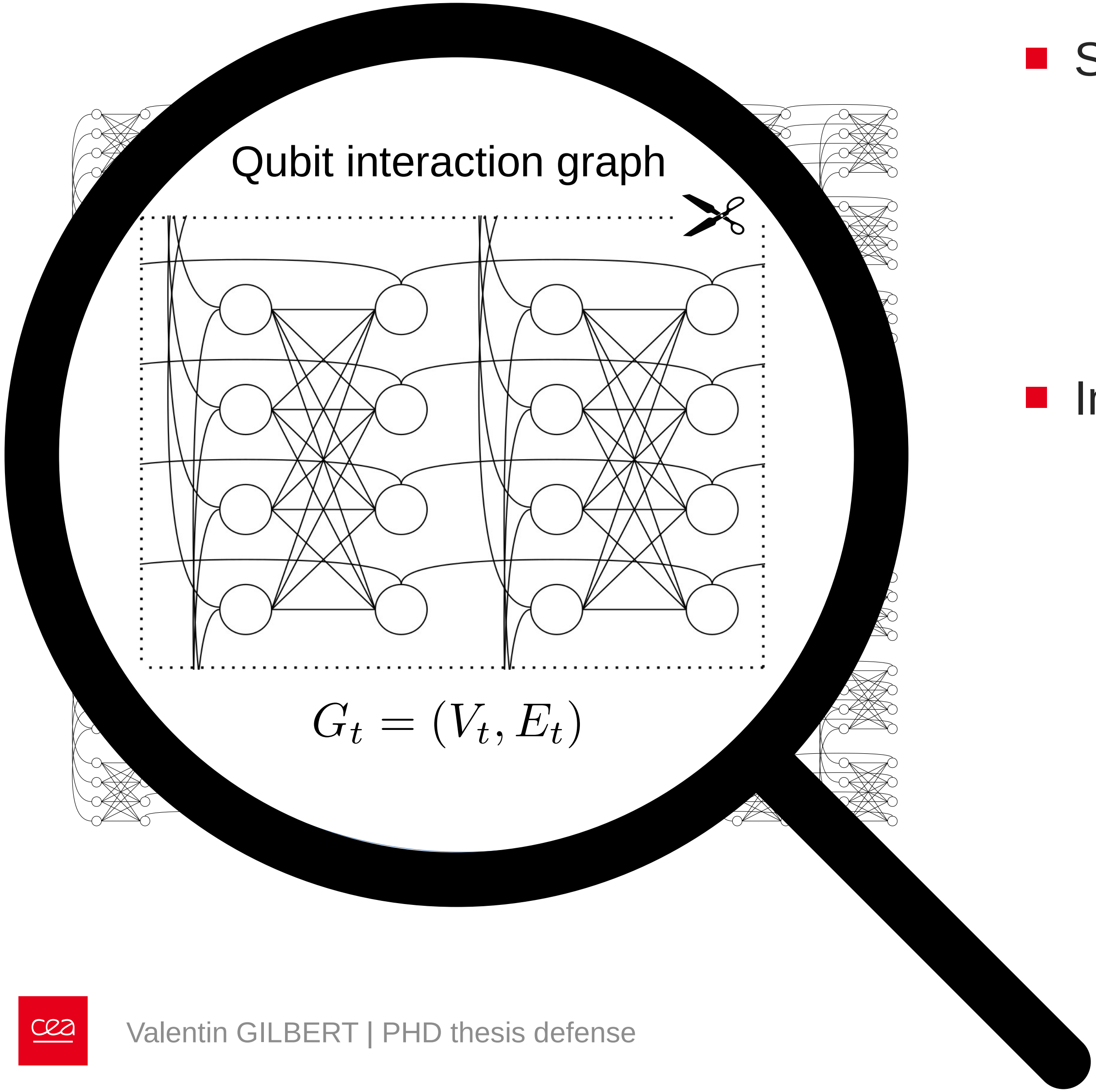


- System Hamiltonian (Transverse Field Ising model)

$$H(s) = -A(s)H_{init} + B(s)H_{final}$$

$$s = \frac{t}{T}, s \in [0, 1]$$

1- Quantum Annealers (D-Wave systems)



- System Hamiltonian (Transverse Field Ising model)

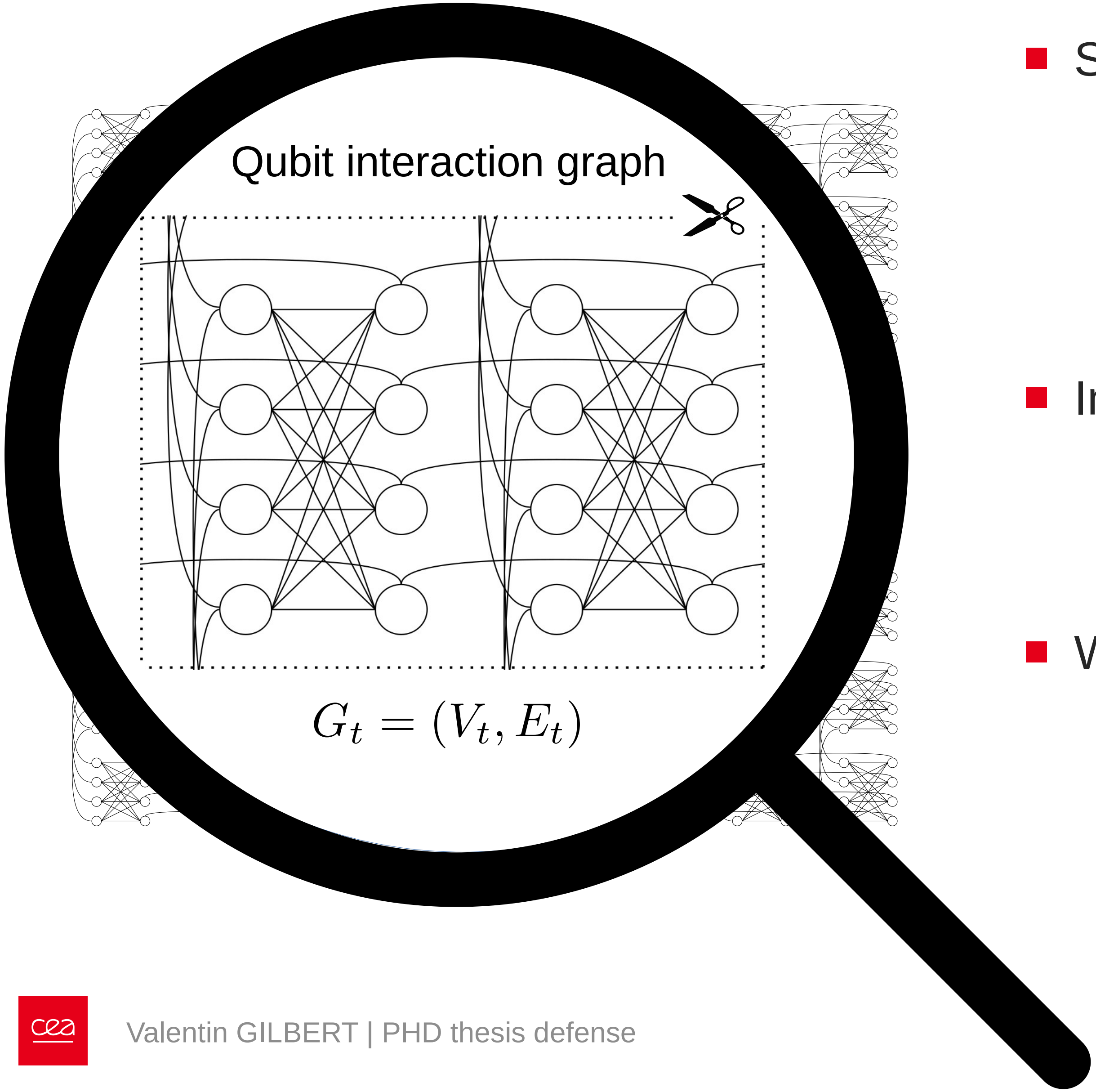
$$H(s) = -A(s)H_{init} + B(s)H_{final}$$

$$s = \frac{t}{T}, s \in [0,1]$$

- Initial Hamiltonian and ground state:

$$H_{init} = \sum_{v \in V_t} \sigma_i^x \qquad \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

1- Quantum Annealers (D-Wave systems)



- System Hamiltonian (Transverse Field Ising model)

$$H(s) = -A(s)H_{init} + B(s)H_{final}$$

$$s = \frac{t}{T}, s \in [0,1]$$

- Initial Hamiltonian and ground state:

$$H_{init} = \sum_{v \in V_t} \sigma_i^x \qquad \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- What eigenstate minimizes the expression $-A(s)\, H_{init}$?

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n - 1} |x\rangle$$

1- Quantum Annealers

- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1- Quantum Annealers

- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

=> $2^{|V_t|}$ eigenvectors and eigenvalues

1- Quantum Annealers

- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

=> $2^{|V_t|}$ eigenvectors and eigenvalues

- Ising cost function correspondence

$$C(s) = \sum_{v \in V_t} h_v s_v + \sum_{(u,v) \in E_t} J_{(u,v)} s_u s_v$$

$$s_v \in \{-1, +1\}$$

1- Quantum Annealers

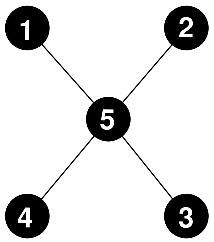
- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

=> $2^{|V_t|}$ eigenvectors and eigenvalues

- Implementation of Maximum Independent Set



$$G_s = (V_s, E_s)$$

- Ising cost function correspondence

$$C(s) = \sum_{v \in V_t} h_v s_v + \sum_{(u,v) \in E_t} J_{(u,v)} s_u s_v$$

$$s_v \in \{-1, +1\}$$

1- Quantum Annealers

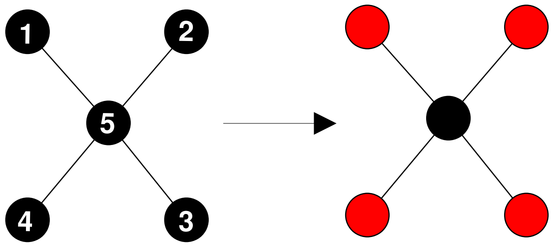
- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

=> $2^{|V_t|}$ eigenvectors and eigenvalues

- Implementation of Maximum Independent Set



$$G_s = (V_s, E_s)$$

- Ising cost function correspondence

$$C(s) = \sum_{v \in V_t} h_v s_v + \sum_{(u,v) \in E_t} J_{(u,v)} s_u s_v$$

$$s_v \in \{-1, +1\}$$

1- Quantum Annealers

- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

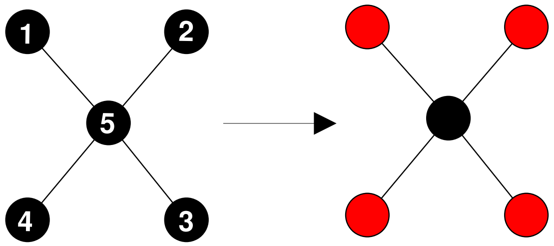
=> $2^{|V_t|}$ eigenvectors and eigenvalues

- Ising cost function correspondence

$$C(s) = \sum_{v \in V_t} h_v s_v + \sum_{(u,v) \in E_t} J_{(u,v)} s_u s_v$$

$$s_v \in \{-1, +1\}$$

- Implementation of Maximum Independent Set



$G_s = (V_s, E_s)$

$$\min - \sum_{v \in V_s} x_v + 2 \sum_{(u,v) \in E_s} x_u x_v$$

1- Quantum Annealers

- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

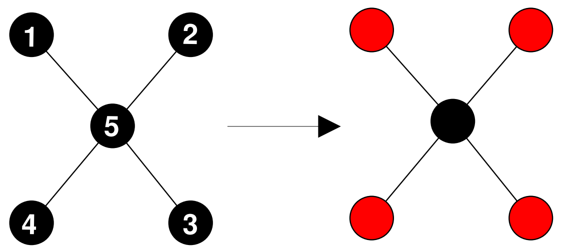
=> $2^{|V_t|}$ eigenvectors and eigenvalues

- Ising cost function correspondence

$$C(s) = \sum_{v \in V_t} h_v s_v + \sum_{(u,v) \in E_t} J_{(u,v)} s_u s_v$$

$$s_v \in \{-1, +1\}$$

- Implementation of Maximum Independent Set



$G_s = (V_s, E_s)$

$$\min - \sum_{v \in V_s} x_v + 2 \sum_{(u,v) \in E_s} x_u x_v$$

$$\min -x_1 - x_2 - x_3 - x_4 - x_5 + 2x_1x_5 + 2x_2x_5 + 2x_3x_5 + 2x_4x_5$$

1- Quantum Annealers

- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

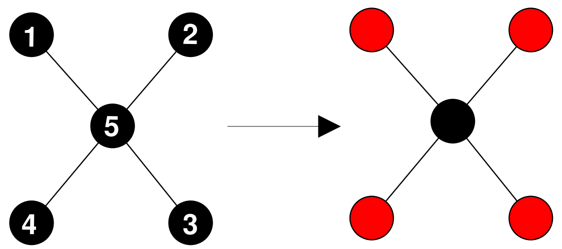
=> $2^{|V_t|}$ eigenvectors and eigenvalues

- Ising cost function correspondence

$$C(s) = \sum_{v \in V_t} h_v s_v + \sum_{(u,v) \in E_t} J_{(u,v)} s_u s_v$$

$$s_v \in \{-1, +1\}$$

- Implementation of Maximum Independent Set



$$G_s = (V_s, E_s)$$

$$\min - \sum_{v \in V_s} x_v + 2 \sum_{(u,v) \in E_s} x_u x_v$$

$$\min -x_1 - x_2 - x_3 - x_4 - x_5 + 2x_1x_5 + 2x_2x_5 + 2x_3x_5 + 2x_4x_5$$

$x_v \to \frac{1 - s_v}{2}$ gives $C(s)$

1- Quantum Annealers

- Problem Hamiltonian

$$H_{final} = \sum_{v \in V_t} h_v \sigma_v^z + \sum_{(u,v) \in E_t} J_{(u,v)} \sigma_u^z \sigma_v^z$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

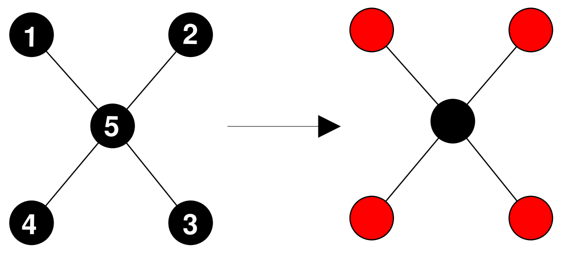
=> $2^{|V_t|}$ eigenvectors and eigenvalues

- Ising cost function correspondence

$$C(s) = \sum_{v \in V_t} h_v s_v + \sum_{(u,v) \in E_t} J_{(u,v)} s_u s_v$$

$$s_v \in \{-1, +1\}$$

- Implementation of Maximum Independent Set



$G_s = (V_s, E_s)$

$$\min - \sum_{v \in V_s} x_v + 2 \sum_{(u,v) \in E_s} x_u x_v$$

$$\min -x_1 - x_2 - x_3 - x_4 - x_5 + 2x_1x_5 + 2x_2x_5 + 2x_3x_5 + 2x_4x_5$$

$x_v \rightarrow \frac{1 - s_v}{2}$ gives $C(s)$

$$H_{final} = -\sigma_5^z + \frac{1}{2}\left(\sigma_1^z\sigma_5^z + \sigma_2^z\sigma_5^z + \sigma_3^z\sigma_5^z + \sigma_4^z\sigma_5^z\right)$$

1- Quantum Annealers

- “Sufficiently slow ?” Analysis of the spectral gap [AL18]:

$$H(s) = -A(s)H_{init} + B(s)H_{final}$$

1- Quantum Annealers

- "Sufficiently slow ?" Analysis of the spectral gap [AL18]:

$$H(s) = -A(s)H_{init} + B(s)H_{final}$$

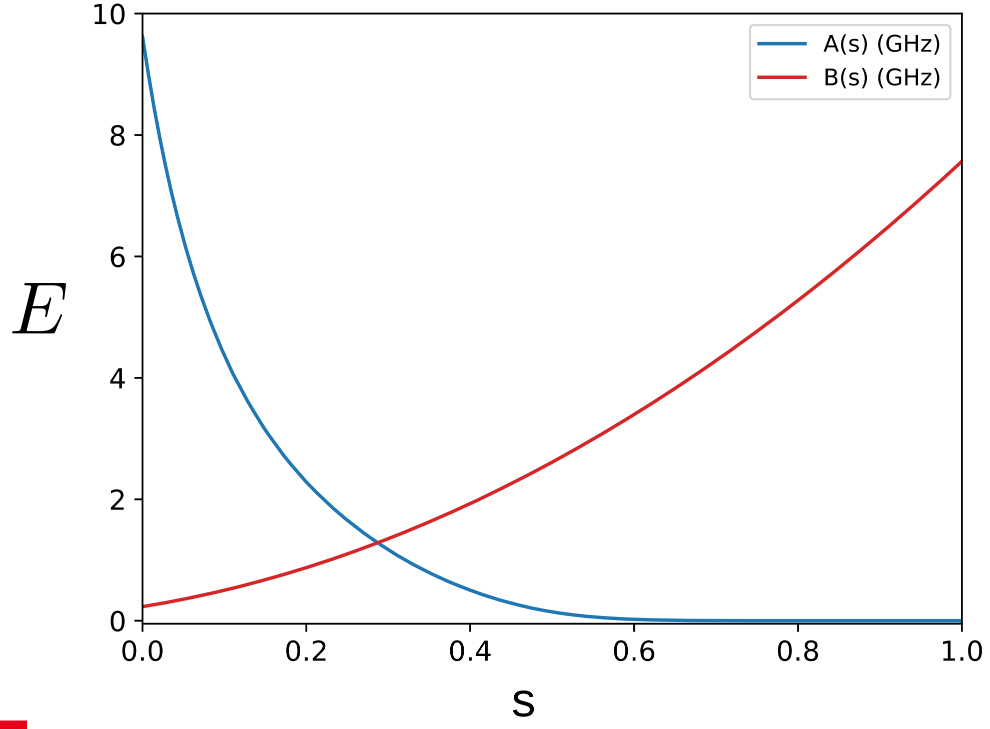
$$H(s)\left|\psi_i(s)\right\rangle = E_i(s)\left|\psi_i(s)\right\rangle, \text{ with } E_0(s) \leq E_1(s) \leq E_2(s) \leq ... \leq E_p(s)$$

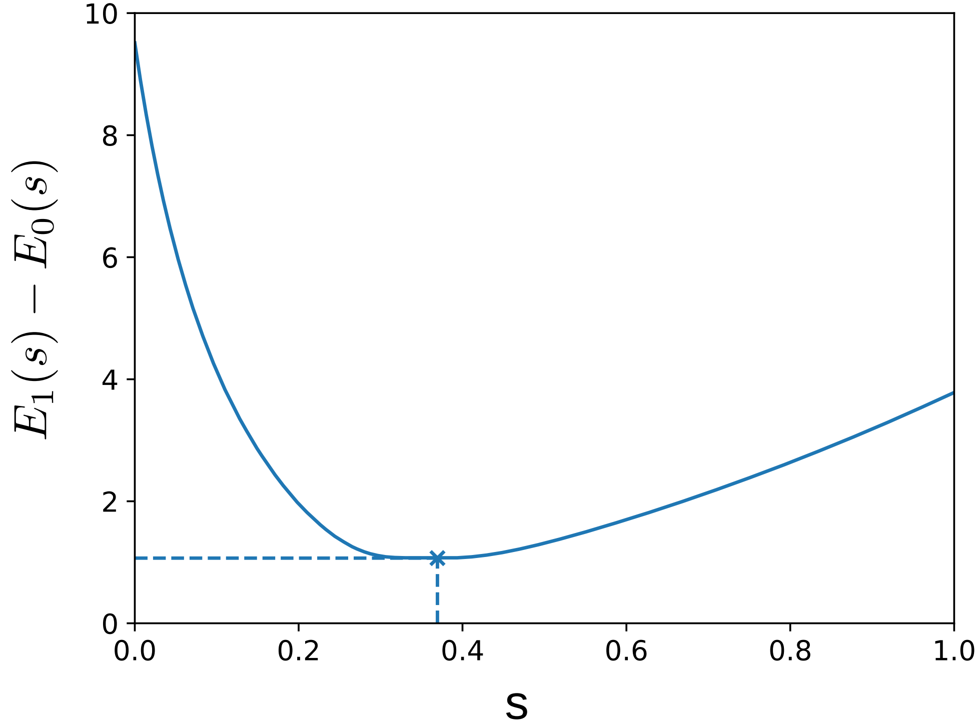
1- Quantum Annealers

- "Sufficiently slow ?" Analysis of the spectral gap [AL18]:

$$H(s) = -A(s)H_{init} + B(s)H_{final}$$

$$H(s)\left|\psi_i(s)\right\rangle = E_i(s)\left|\psi_i(s)\right\rangle, \text{ with } E_0(s) \leq E_1(s) \leq E_2(s) \leq ... \leq E_p(s)$$





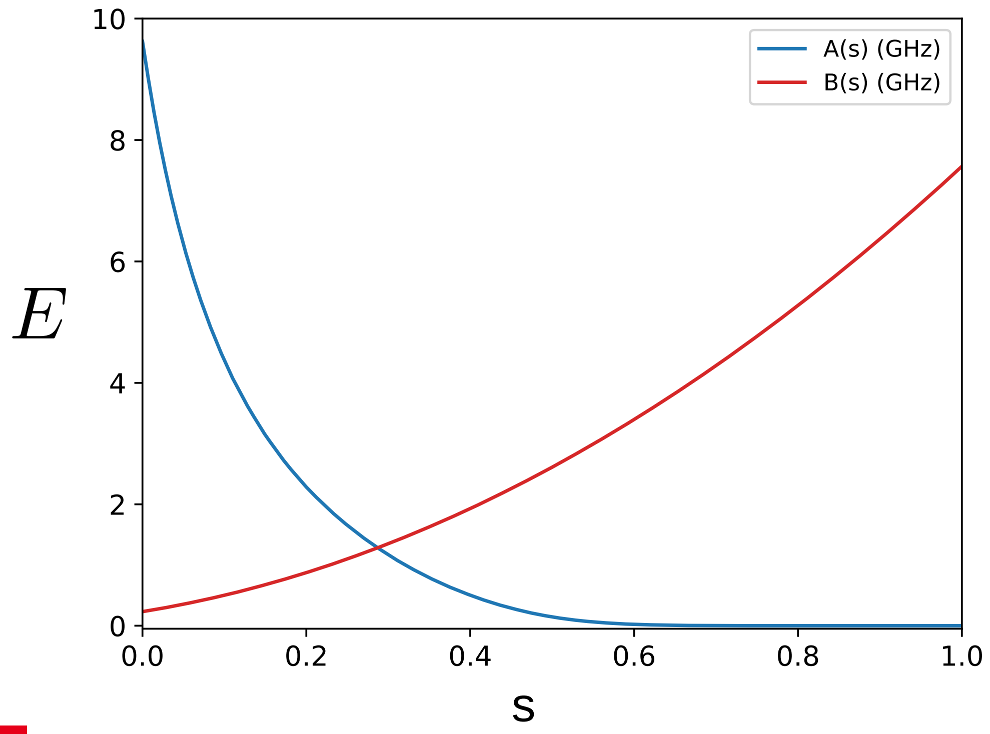
1- Quantum Annealers

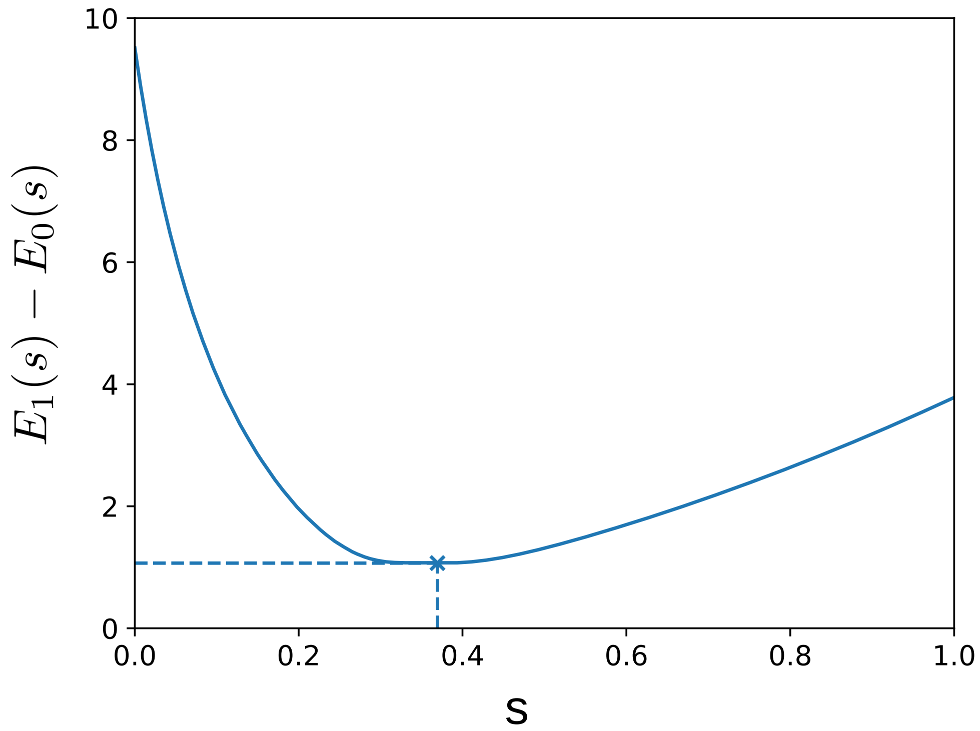
- "Sufficiently slow ?" Analysis of the spectral gap [AL18]:

$$H(s) = -A(s)H_{init} + B(s)H_{final}$$

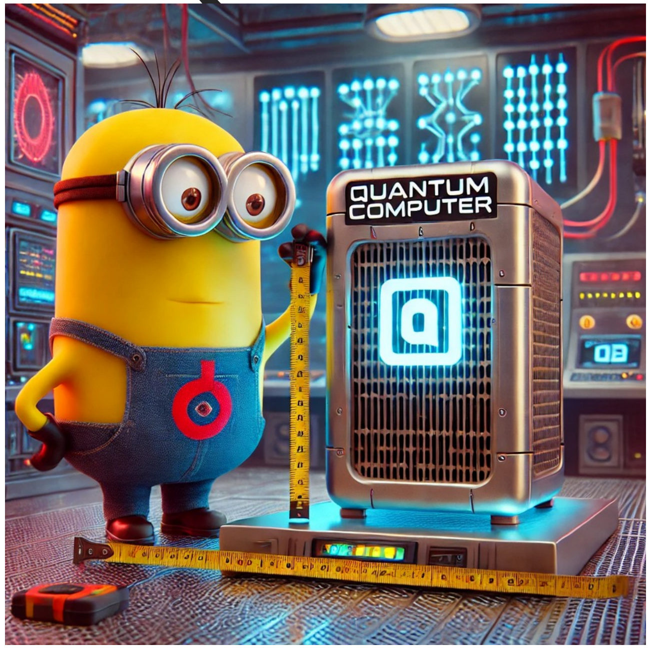
$$H(s)\left|\psi_i(s)\right\rangle = E_i(s)\left|\psi_i(s)\right\rangle, \text{ with } E_0(s) \leq E_1(s) \leq E_2(s) \leq \ldots \leq E_p(s)$$

$$g_{\min} = \min_{0\leq s\leq 1} E_1(s) - E_0(s) \qquad T \propto O(1/g_{\min}^2)$$





2. Contribution #1 Performance evaluation of Quantum Annealers



Generated with openai

2- Performance evaluation of Quantum Annealers

Application benchmark instances

Random instances :
+ Generate large sets of instances
- Relevance of the set can be debated
- Phase transition issue for some instances

Crafted instances :
+ Easy to find instances hard for classical solvers
+ Create instances with planted solutions
- Some classical methods may take advantage of the structure of the instance

Real-world instances :
+ Ideal targets
- Hard to design due to quantum hardware limited size
- Hard to find by nature (selection bias)

2- Performance evaluation of Quantum Annealers

Application benchmark instances

Random instances :
+ Generate large sets of instances
- Relevance of the set can be debated
- Phase transition issue for some instances

Crafted instances :
+ Easy to find instances hard for classical solvers
+ Create instances with planted solutions
- Some classical methods may take advantage of the structure of the instance

Real-world instances :
+ Ideal targets
- Hard to design due to quantum hardware limited size
- Hard to find by nature (selection bias)

2- Performance evaluation of Quantum Annealers

Application benchmark instances

Random instances :
+ Generate large sets of instances
- Relevance of the set can be debated
- Phase transition issue for some instances

Crafted instances :
+ Easy to find instances hard for classical solvers
+ Create instances with planted solutions
- Some classical methods may take advantage of the structure of the instance

Real-world instances :
+ Ideal targets
- Hard to design due to quantum hardware limited size
- Hard to find by nature (selection bias)

Source	Instance topology	#Variables	QPU	Embedding
2021 [PCL+21]	Chimera graph	2032	2000Q	QPU chip sub-graph
2022 [TAM+22]	Pegasus graph	5387	Adv4.1	QPU chip-subgraph
2023 [LCG+24]	3-regular graph	4-320	Adv4.1	QPU chip-subgraph
2024 [KNR+24]	Square, cubic, Diamond, biclique	16-567	Adv4.1 Adv2	QPU chip-subgraph 2qubits/var

2- Performance evaluation of Quantum Annealers

Application benchmark instances

Random instances :
+ Generate large sets of instances
- Relevance of the set can be debated
- Phase transition issue for some instances

Crafted instances :
+ Easy to find instances hard for classical solvers
+ Create instances with planted solutions
- Some classical methods may take advantage of the structure of the instance

Real-world instances :
+ Ideal targets
- Hard to design due to quantum hardware limited size
- Hard to find by nature (selection bias)

Source	Instance topology	#Variables	QPU	Embedding
2021 [PCL+21]	Chimera graph	2032	2000Q	QPU chip sub-graph
2022 [TAM+22]	Pegasus graph	5387	Adv4.1	QPU chip-subgraph
2023 [LCG+24]	3-regular graph	4-320	Adv4.1	QPU chip-subgraph
2024 [KNR+24]	Square, cubic, Diamond, biclique	16-567	Adv4.1 Adv2	QPU chip-subgraph 2qubits/var

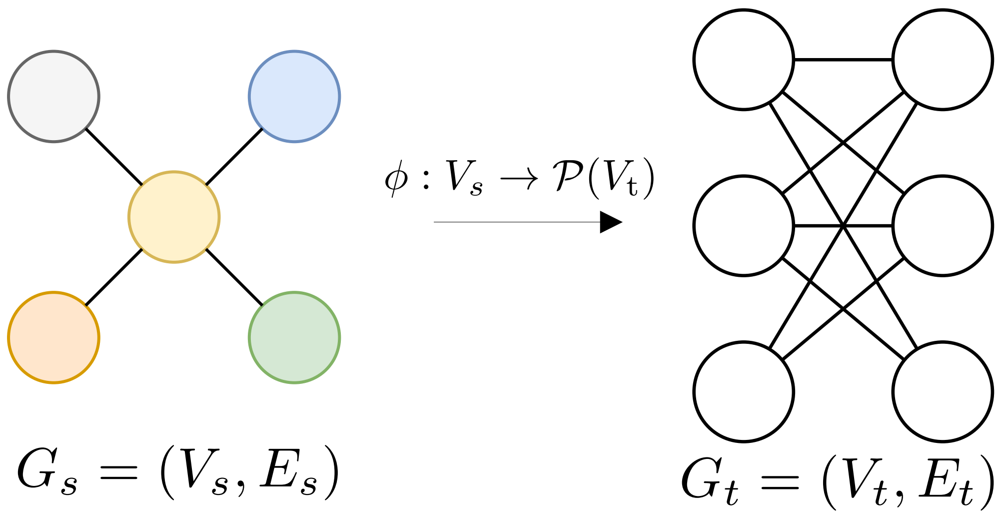
2- Generation of instances

- Minor-embedding (Graph Minor Theory [RS95])

$G_s = (V_s, E_s)$ $\qquad$ $G_t = (V_t, E_t)$

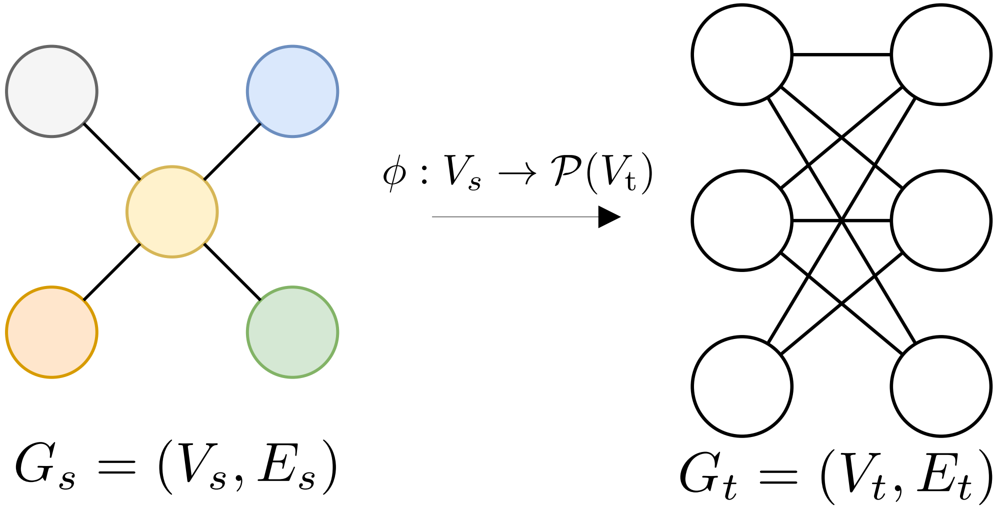
2- Generation of instances

- Minor-embedding (Graph Minor Theory [RS95])

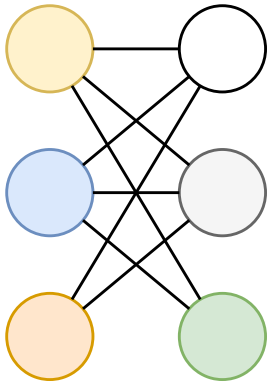


2- Generation of instances

- Minor-embedding (Graph Minor Theory [RS95])

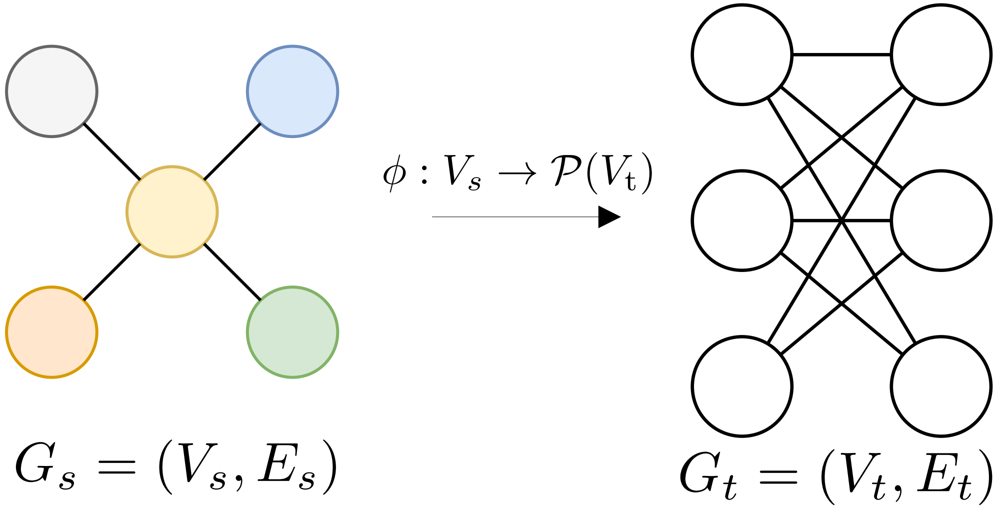


<u>Rule 1:</u>
All the edges in G_s must be represented in G_t

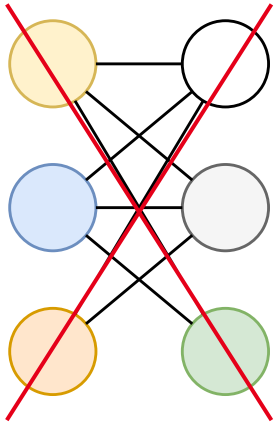


2- Generation of instances

- Minor-embedding (Graph Minor Theory [RS95])

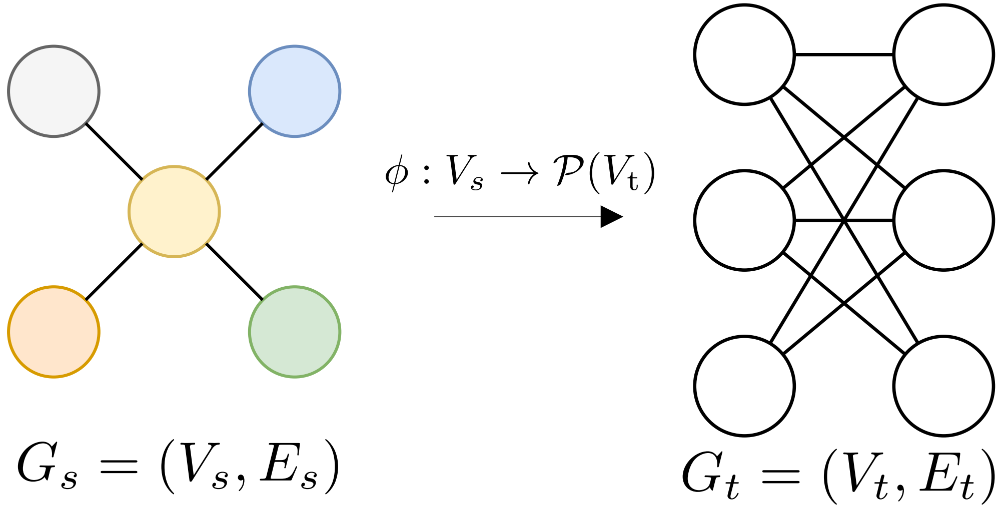


<u>Rule 1:</u>
All the edges in G_s must be represented in G_t

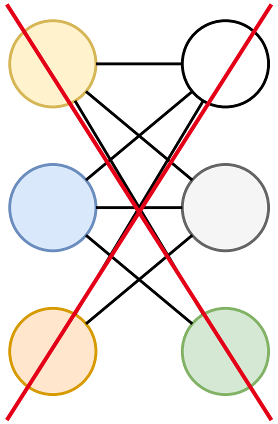


2- Generation of instances

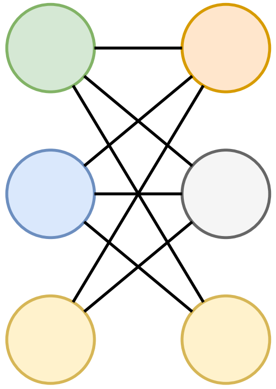
- Minor-embedding (Graph Minor Theory [RS95])



Rule 1:
All the edges in G_s must be represented in G_t

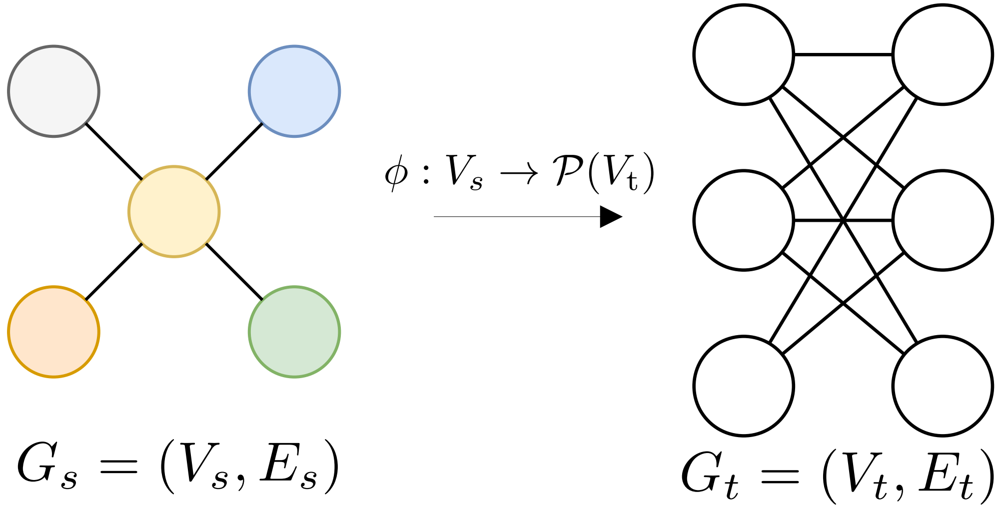


Rule 2:
A logical qubit is represented by a connected subgraph of physical qubits

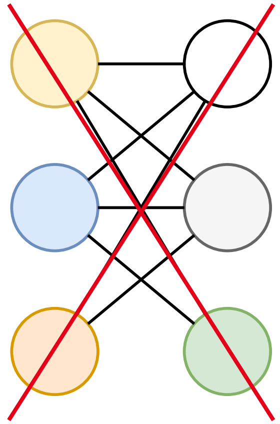


2- Generation of instances

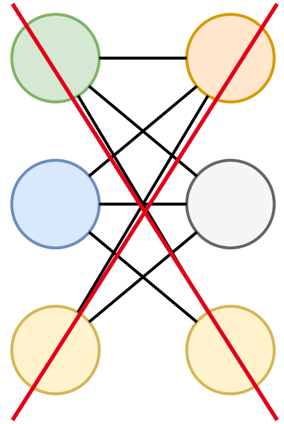
- Minor-embedding (Graph Minor Theory [RS95])



Rule 1:
All the edges in G_s must be represented in G_t

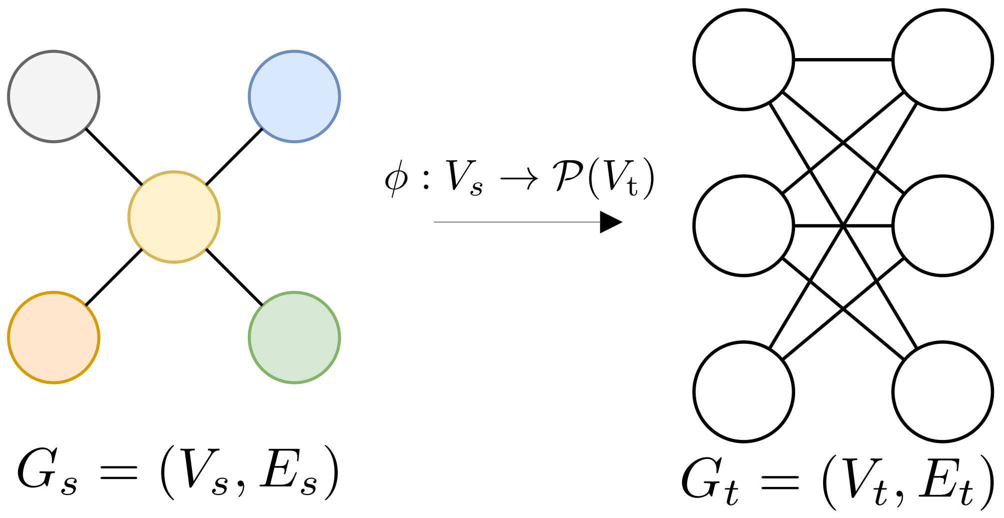


Rule 2:
A logical qubit is represented by a connected subgraph of physical qubits

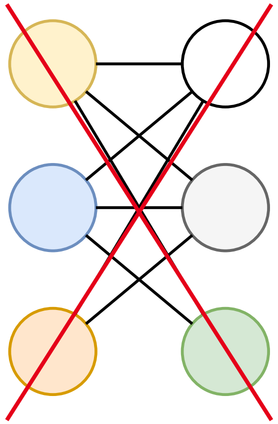


2- Generation of instances

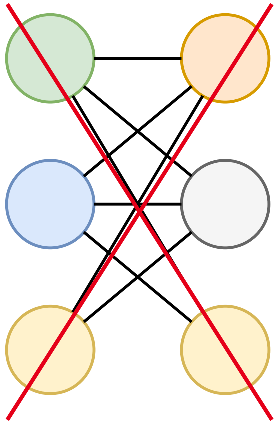
- Minor-embedding (Graph Minor Theory [RS95])



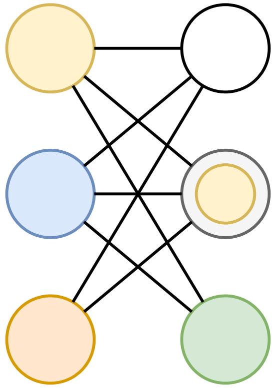
Rule 1:
All the edges in G_s must be represented in G_t



Rule 2:
A logical qubit is represented by a connected subgraph of physical qubits

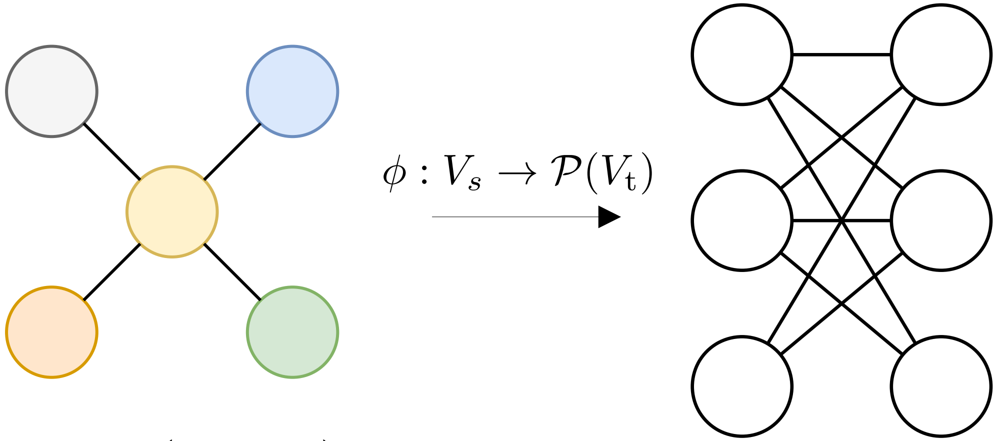


Rule 3:
No overlap



2- Generation of instances

- Minor-embedding (Graph Minor Theory [RS95])

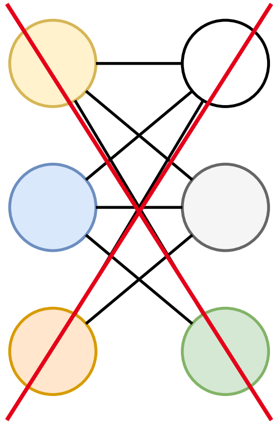


$\phi : V_s \rightarrow \mathcal{P}(V_t)$

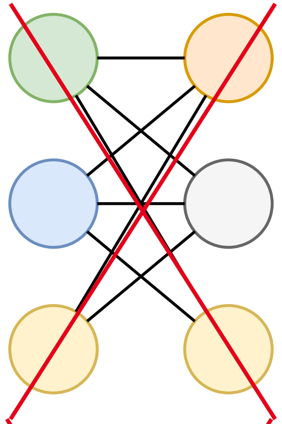
$G_s = (V_s, E_s)$

$G_t = (V_t, E_t)$

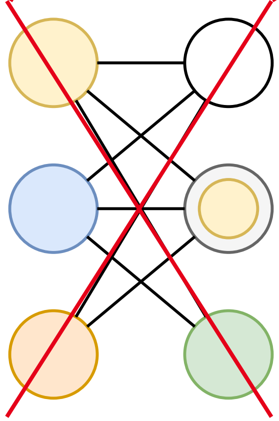
<u>Rule 1:</u>
All the edges in G_s must be represented in G_t



<u>Rule 2:</u>
A logical qubit is represented by a connected subgraph of physical qubits

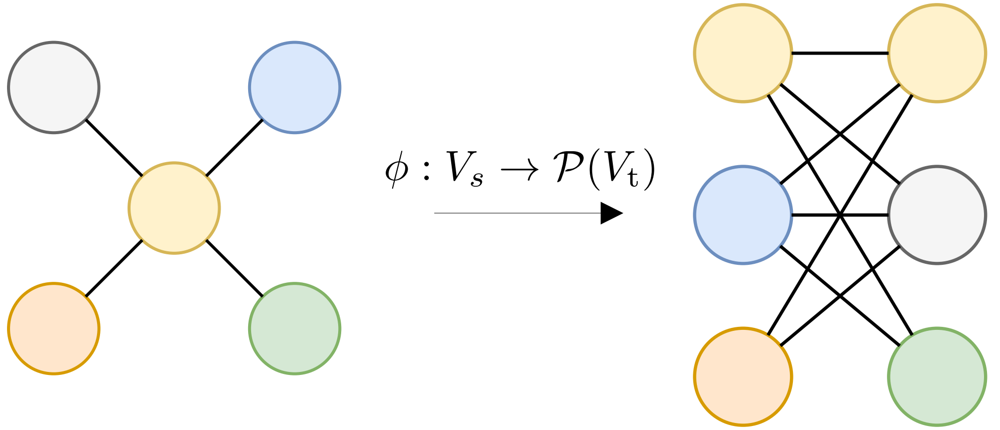


<u>Rule 3:</u>
No overlap



2- Generation of instances

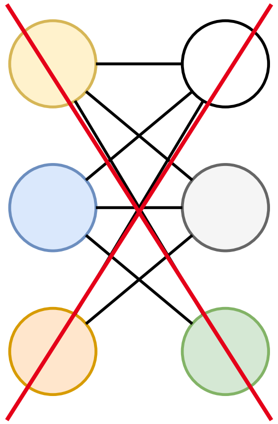
- Minor-embedding (Graph Minor Theory [RS95])



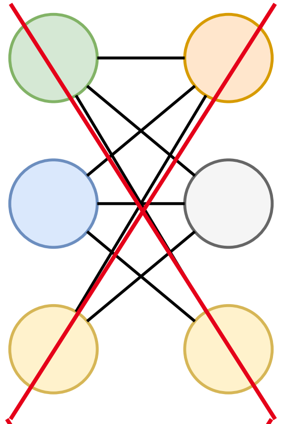
$G_s = (V_s, E_s)$

$G_t = (V_t, E_t)$

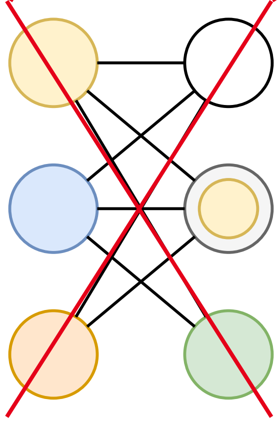
<u>Rule 1:</u>
All the edges in G_s must be represented in G_t



<u>Rule 2:</u>
A logical qubit is represented by a connected subgraph of physical qubits

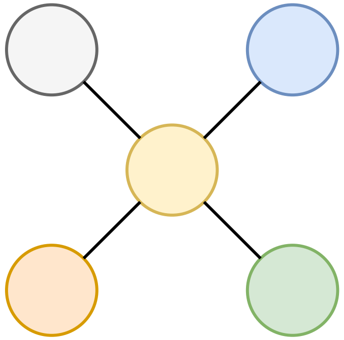


<u>Rule 3:</u>
No overlap



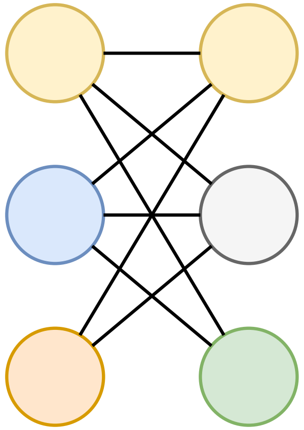
2- Generation of instances

- Minor-embedding (Graph Minor Theory [RS95])



$G_s = (V_s, E_s)$

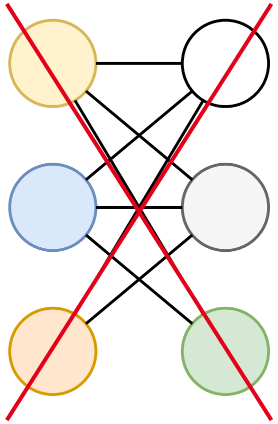
$\phi : V_s \rightarrow \mathcal{P}(V_t)$



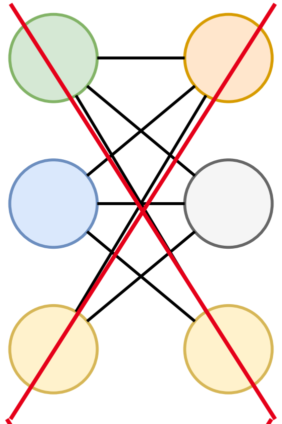
$G_t = (V_t, E_t)$

NP-Hard problem for arbitrary graphs

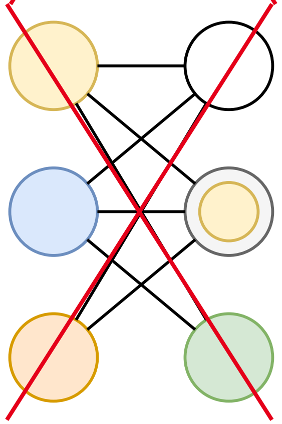
Rule 1:
All the edges in G_s must be represented in G_t



Rule 2:
A logical qubit is represented by a connected subgraph of physical qubits

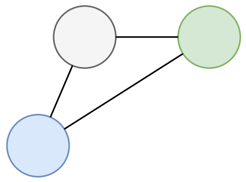


Rule 3:
No overlap

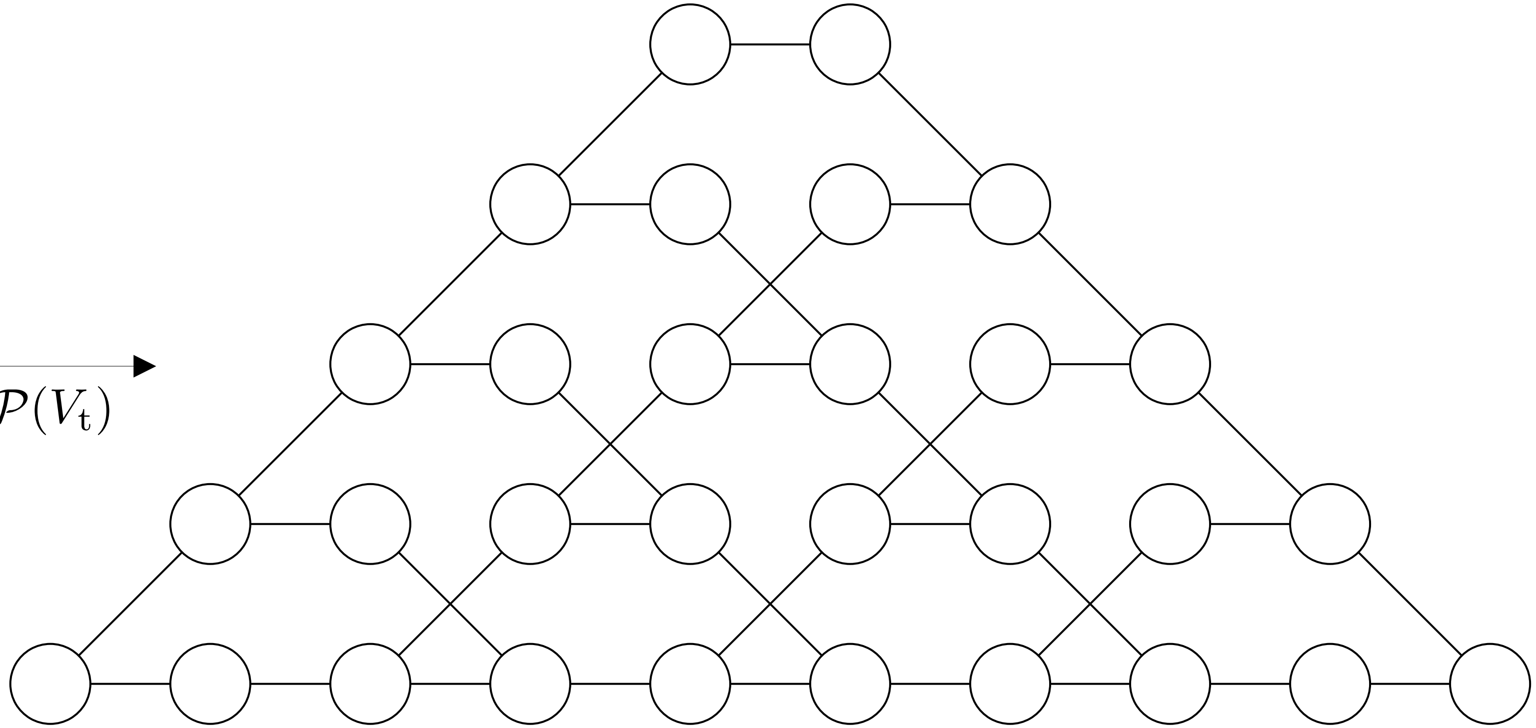


2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]

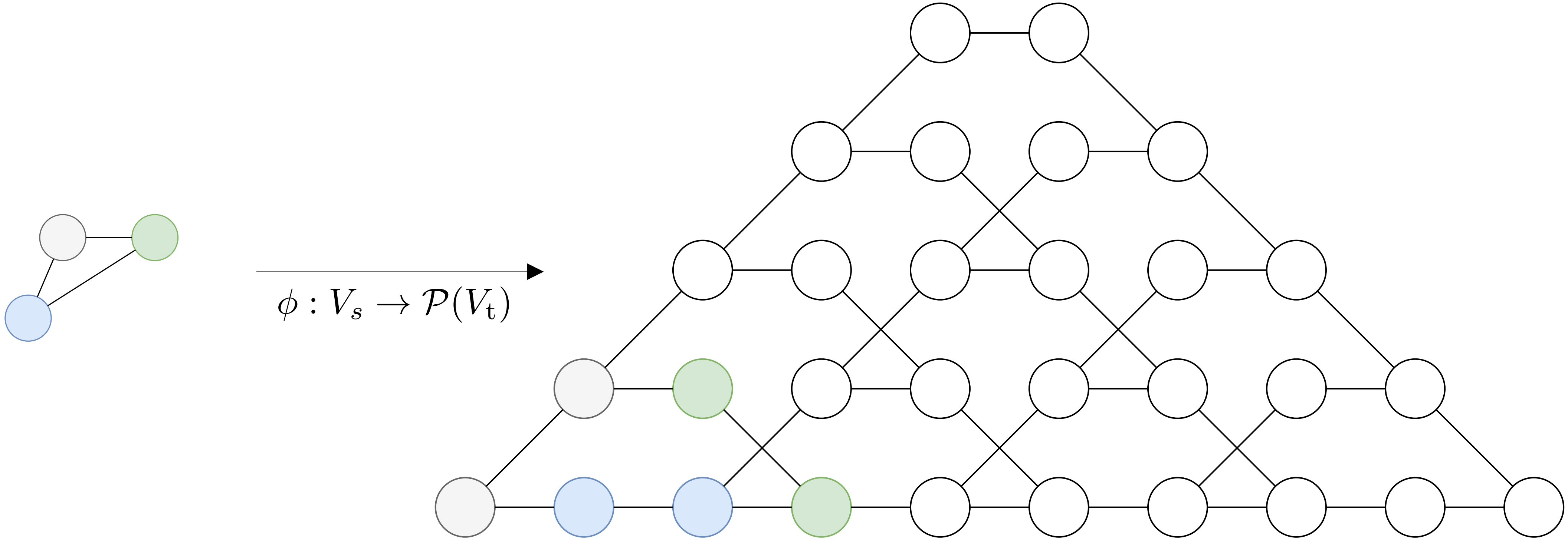


$$\phi : V_s \rightarrow \mathcal{P}(V_t)$$



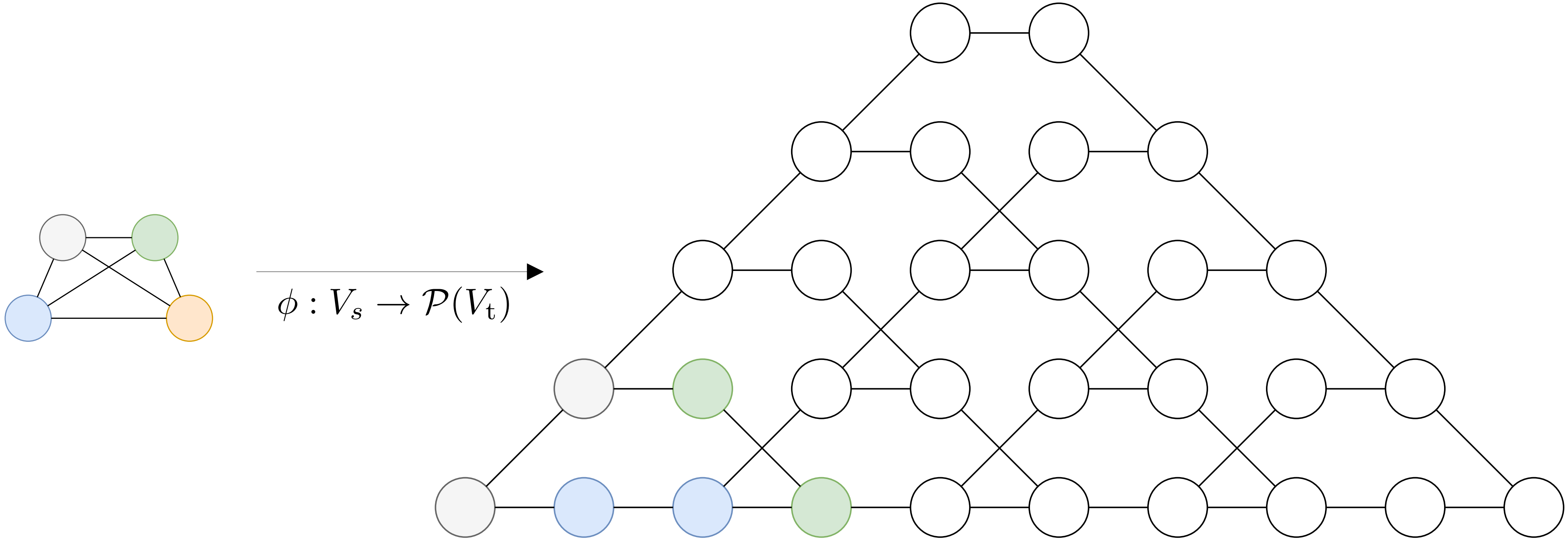
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]



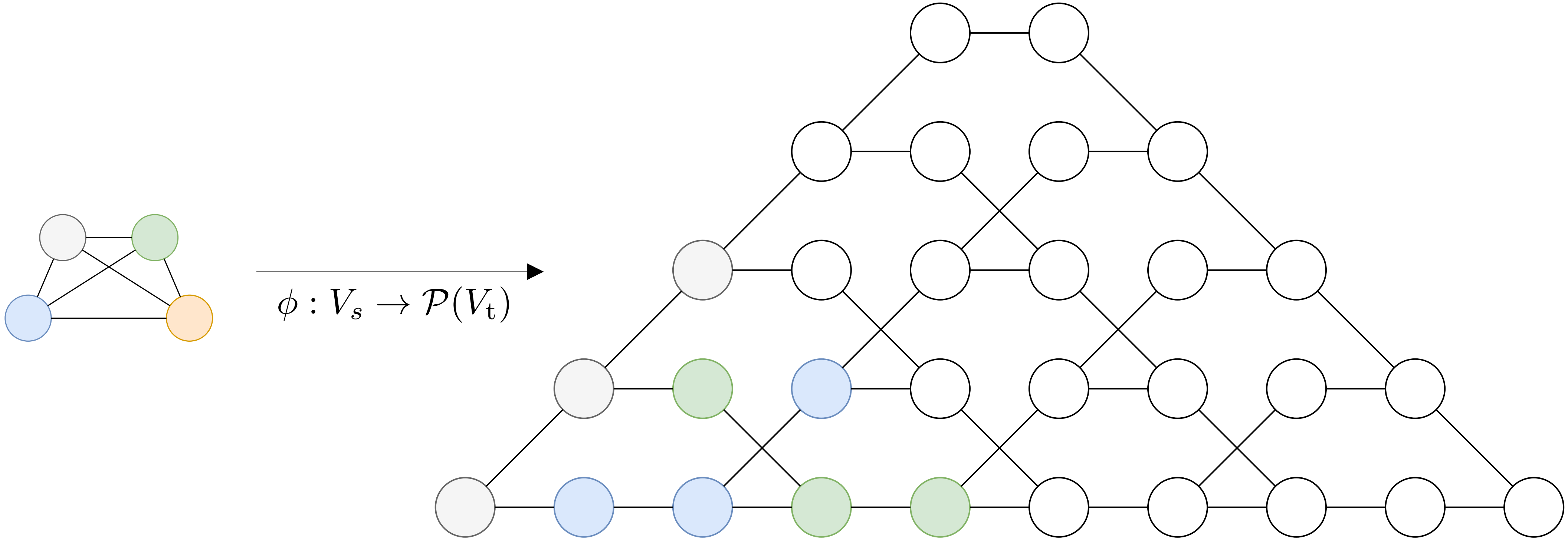
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]



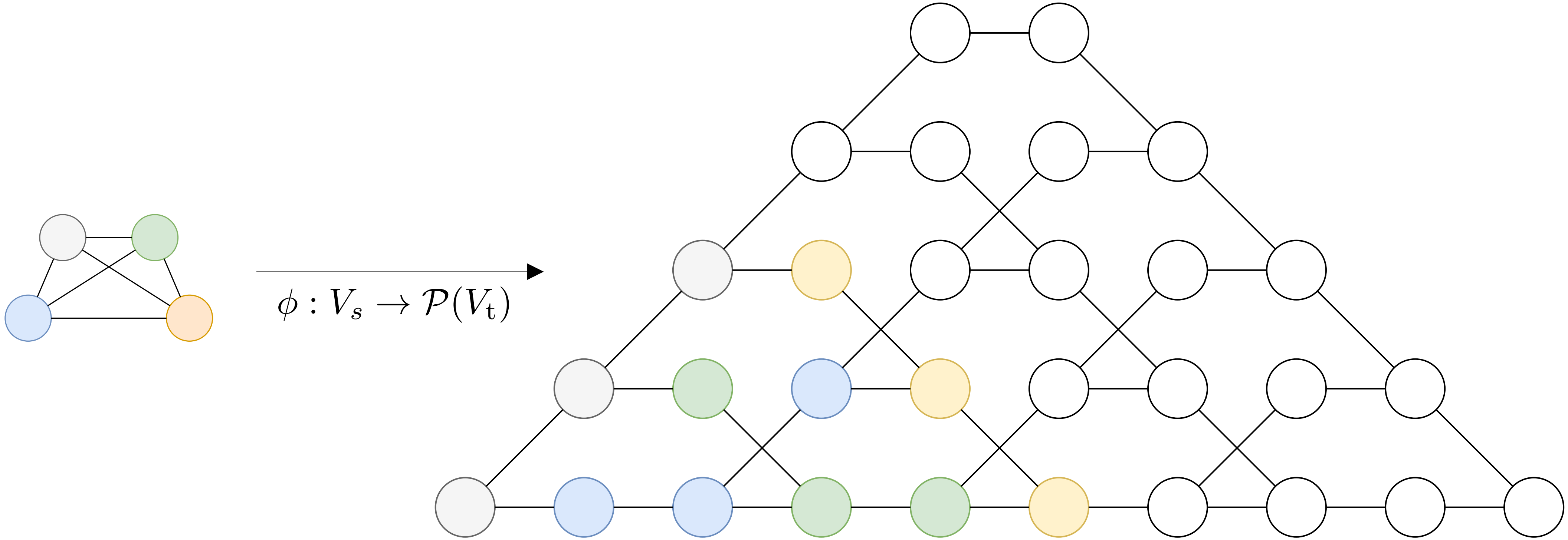
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]



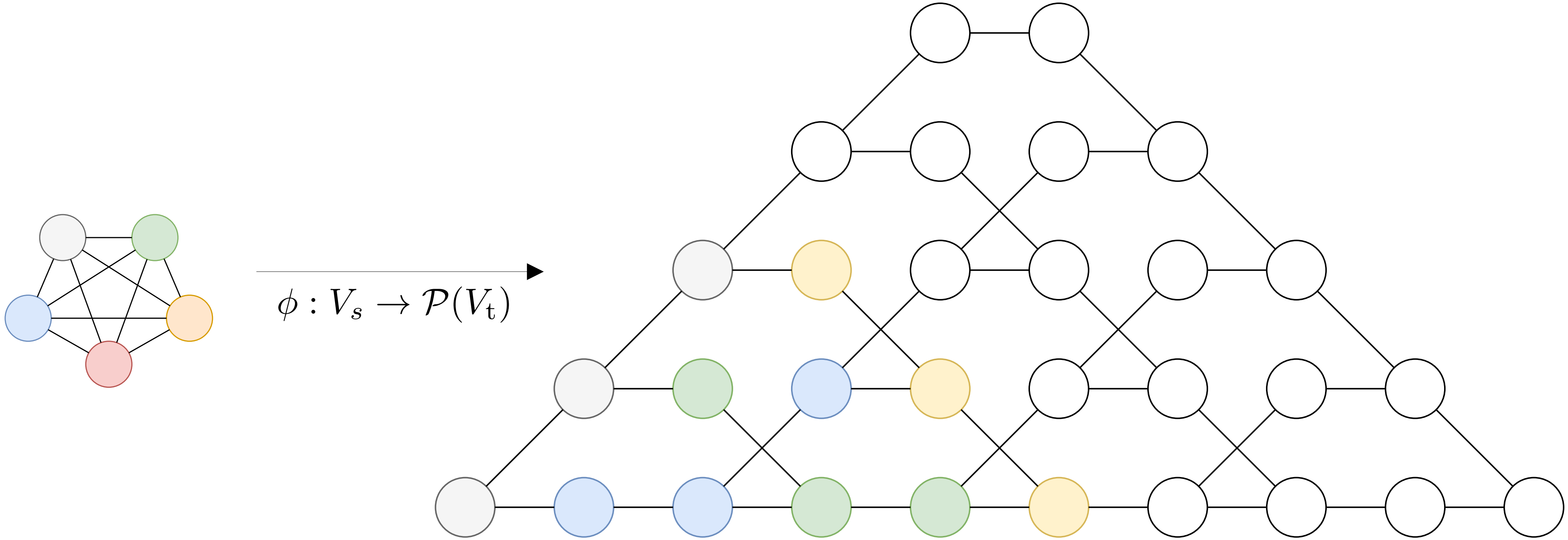
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]



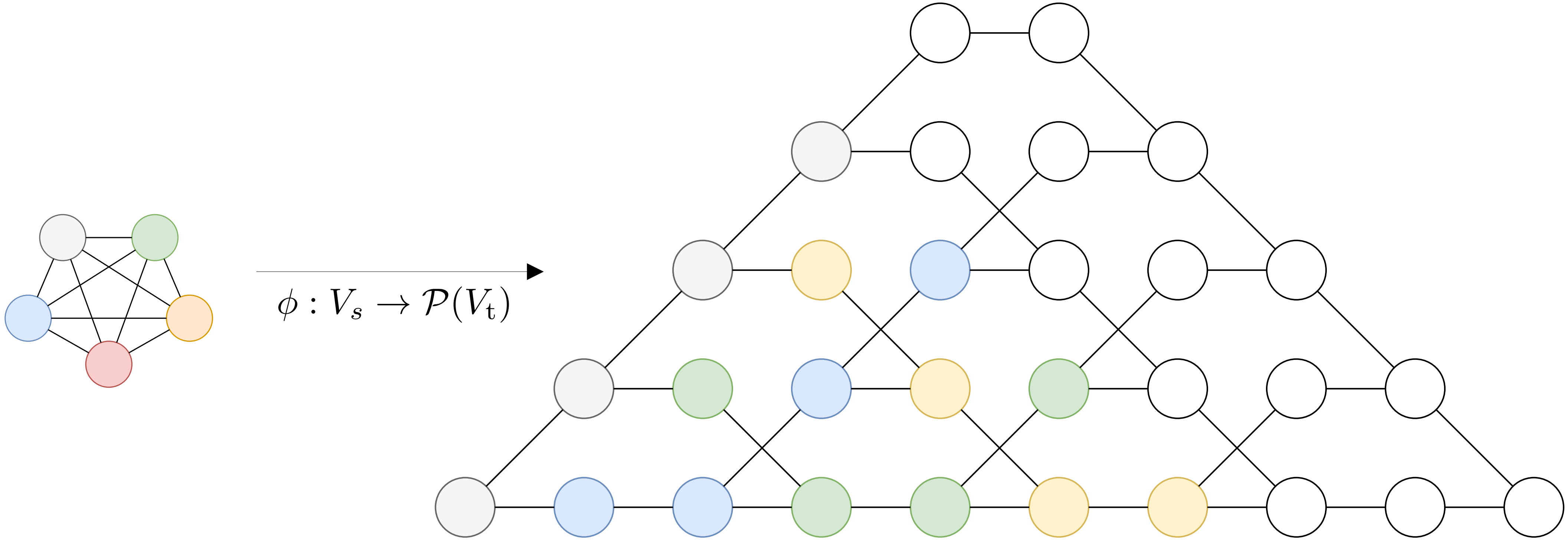
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]



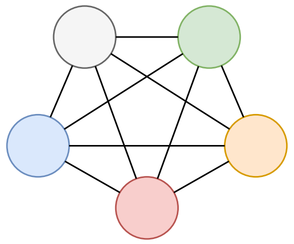
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]

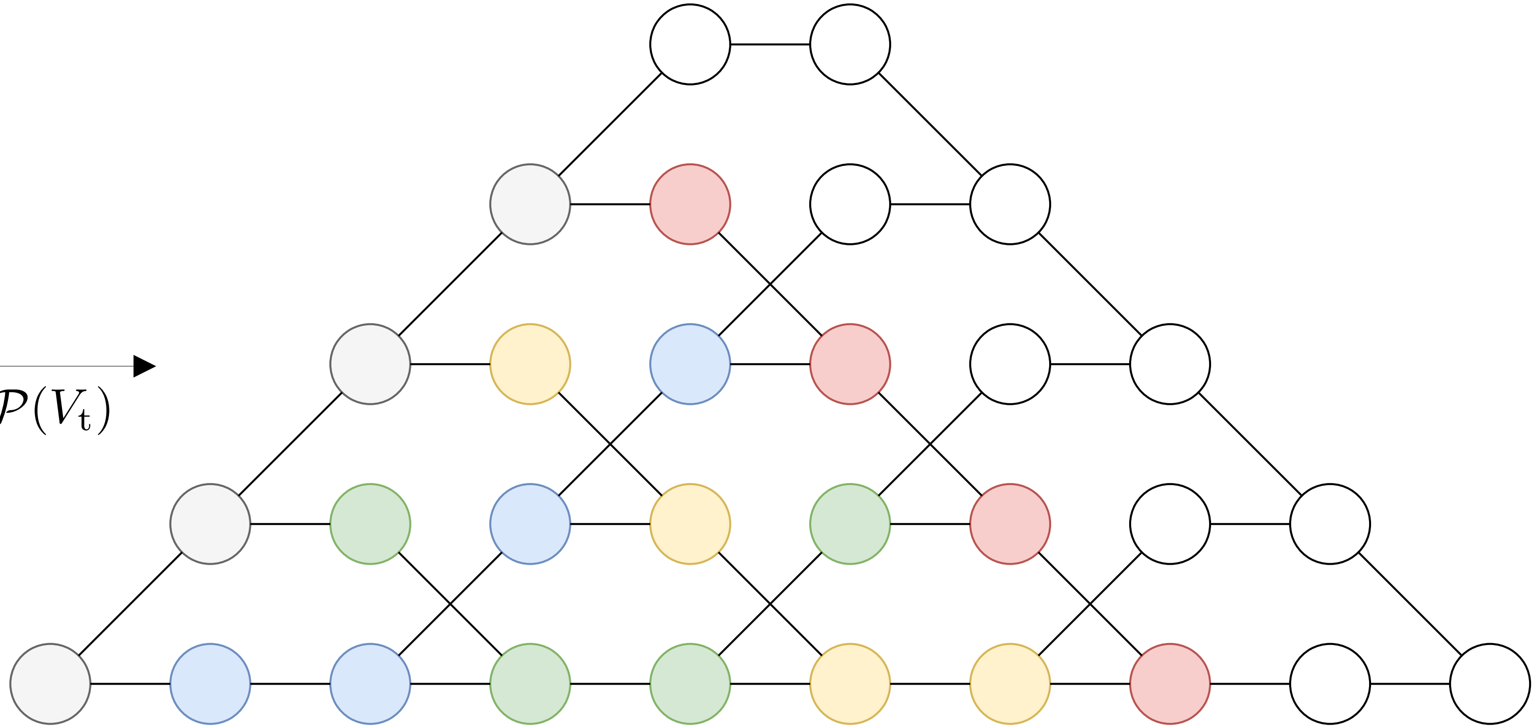


2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern) [Cho11]

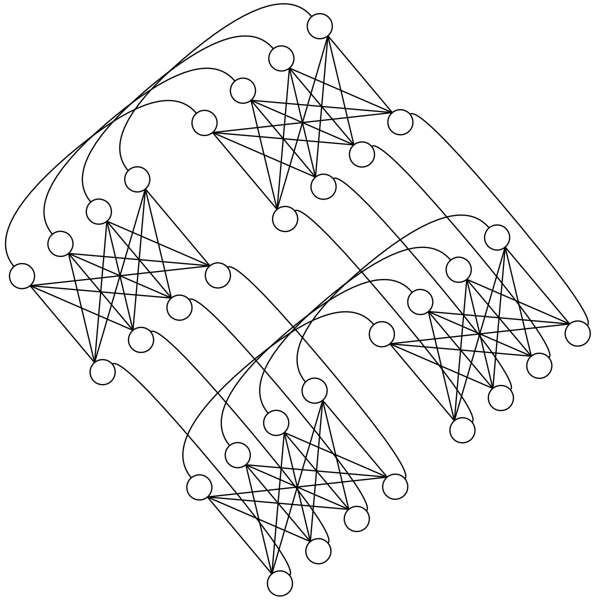


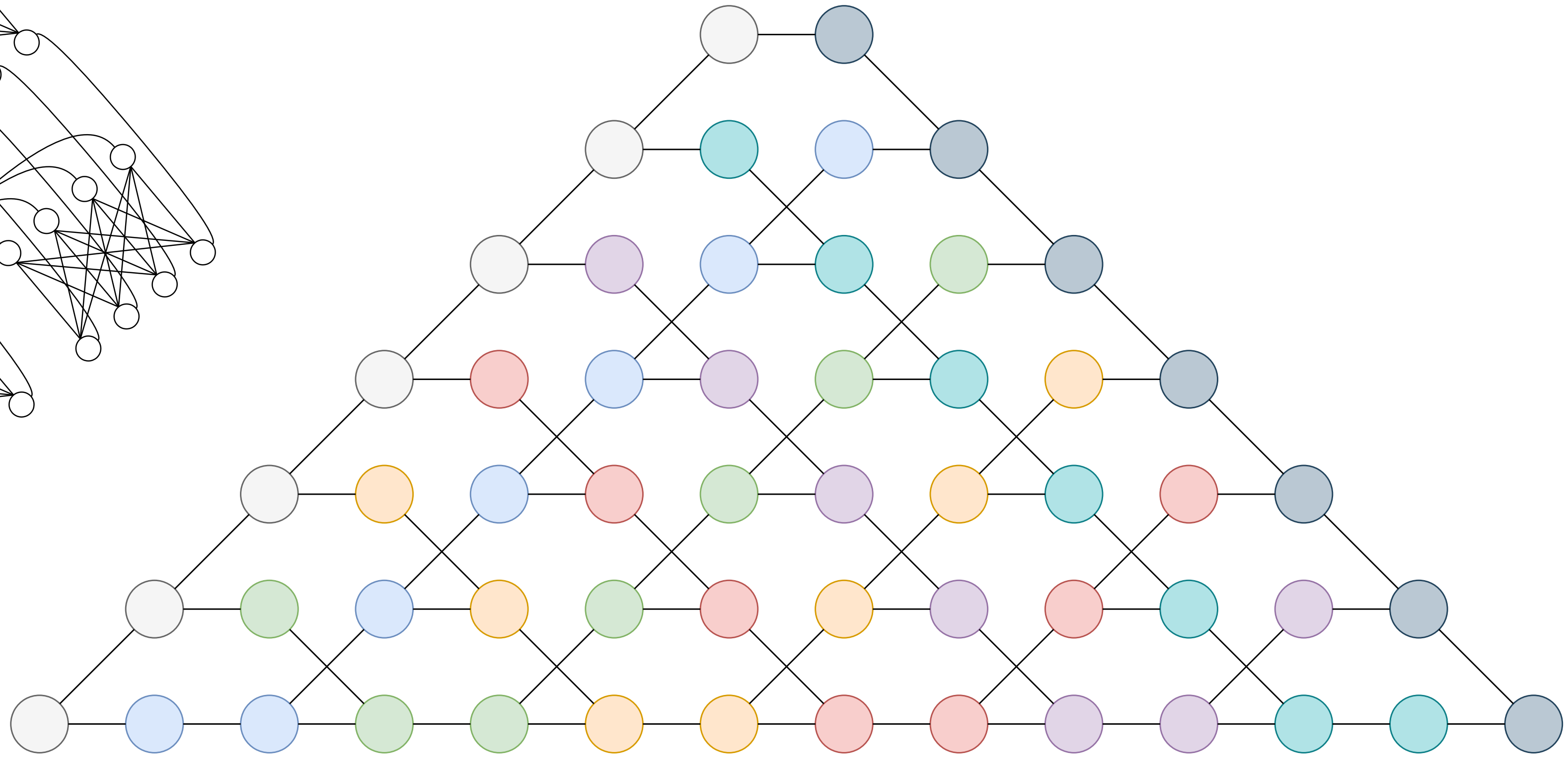
$$\phi : V_s \rightarrow \mathcal{P}(V_t)$$



2- Generation of instances

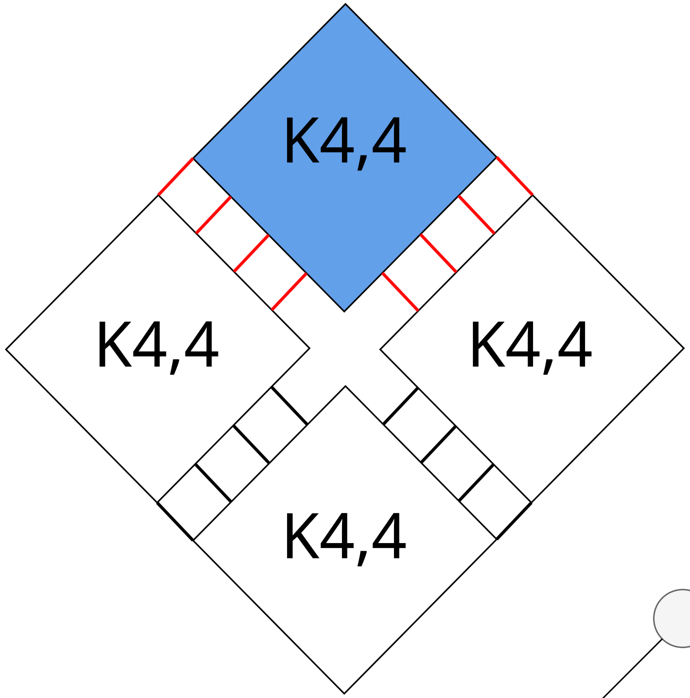
- Embedding of complete graphs (TRIAD Pattern)

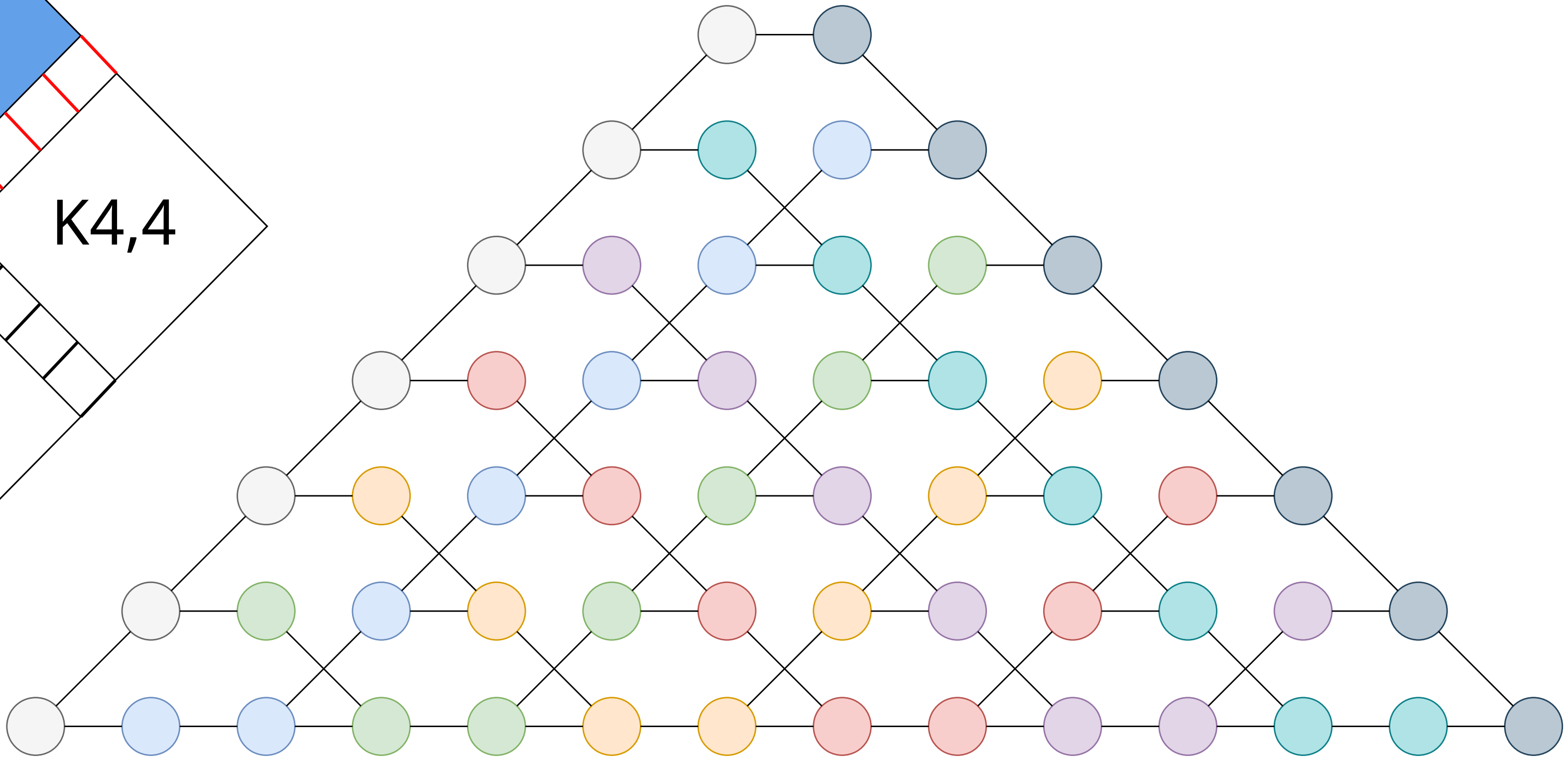




2- Generation of instances

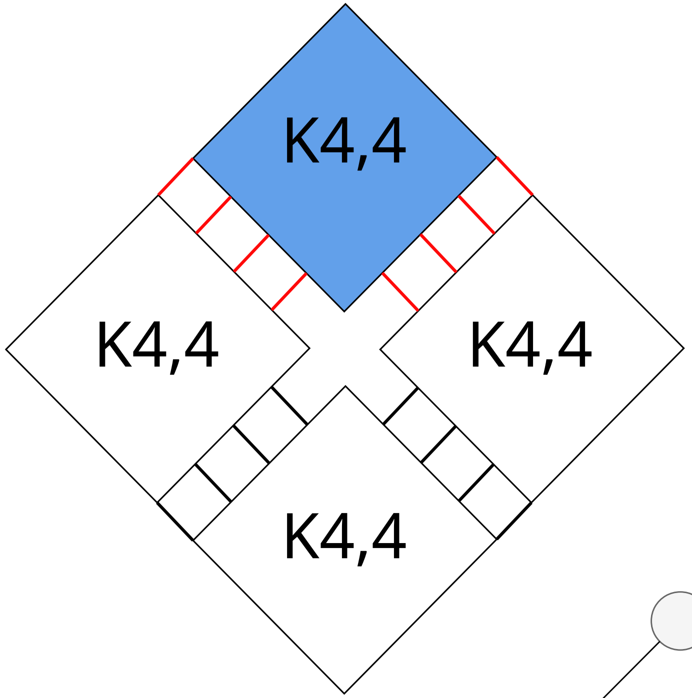
- Embedding of complete graphs (TRIAD Pattern)

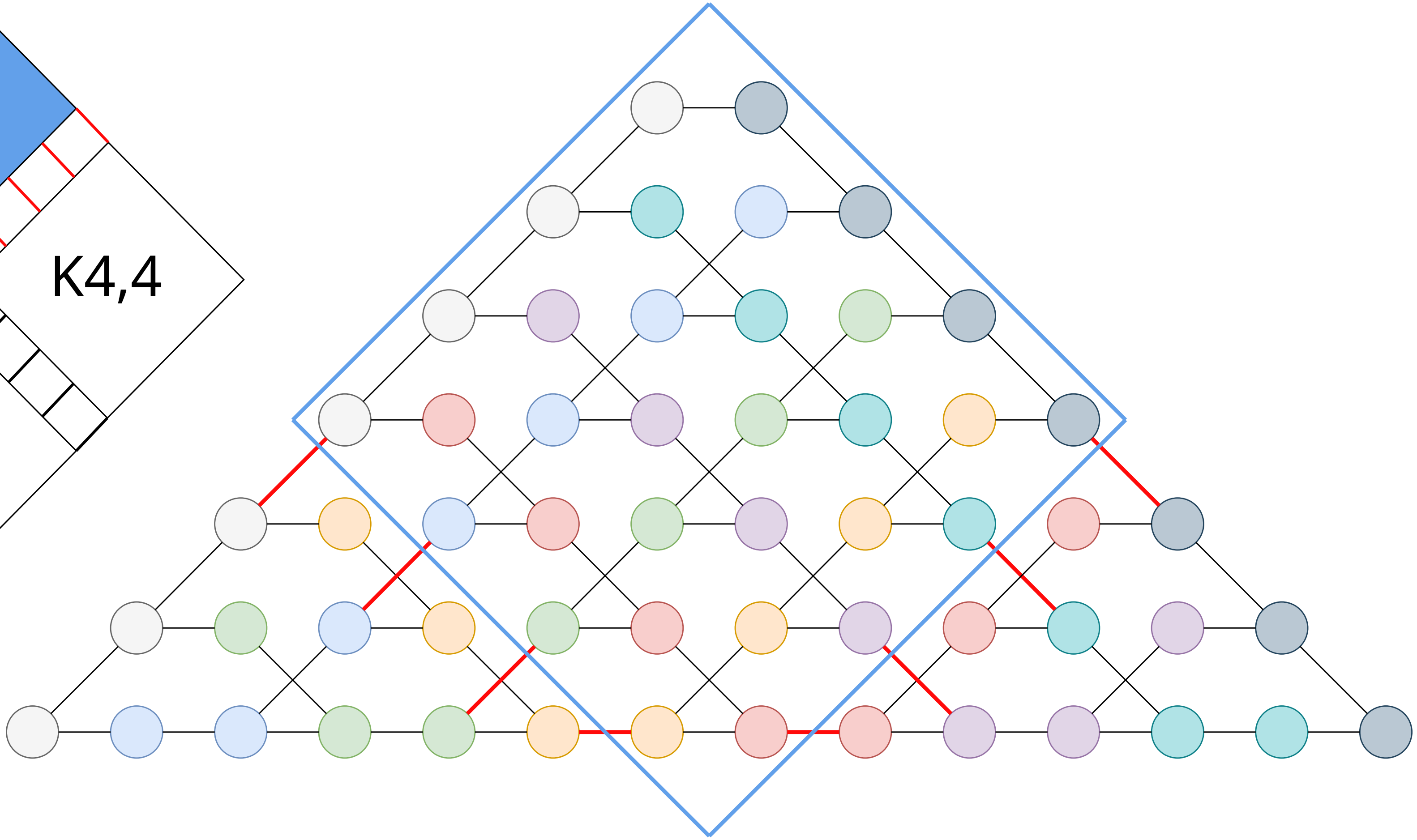




2- Generation of instances

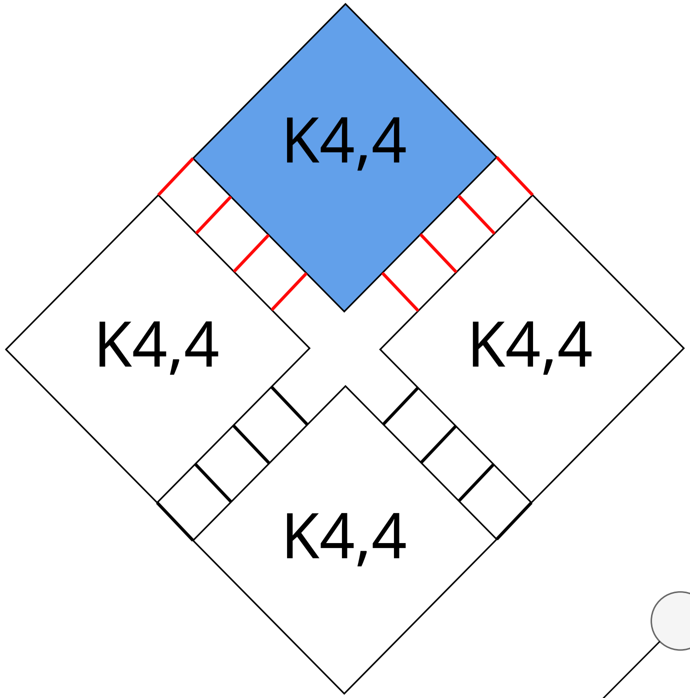
- Embedding of complete graphs (TRIAD Pattern)

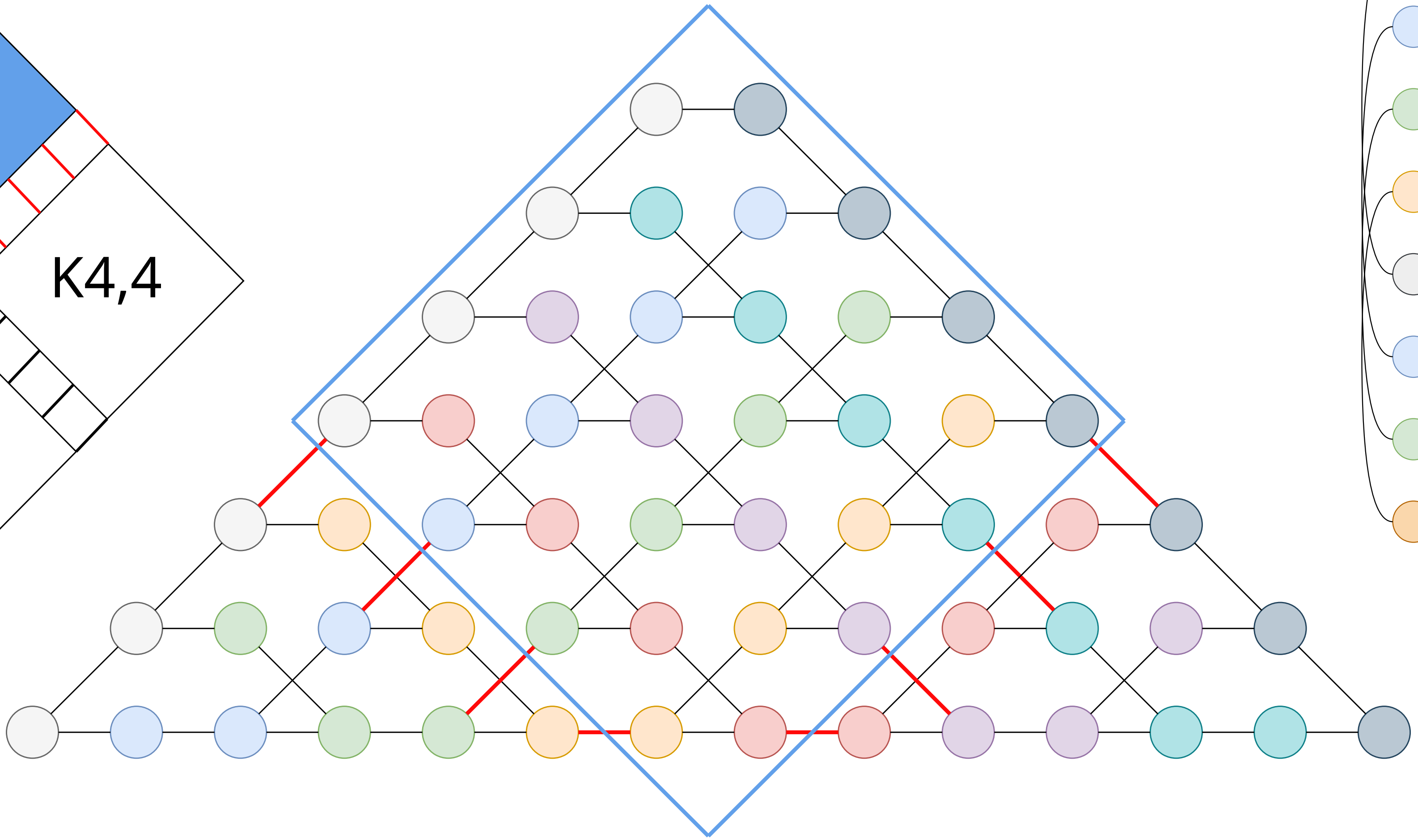


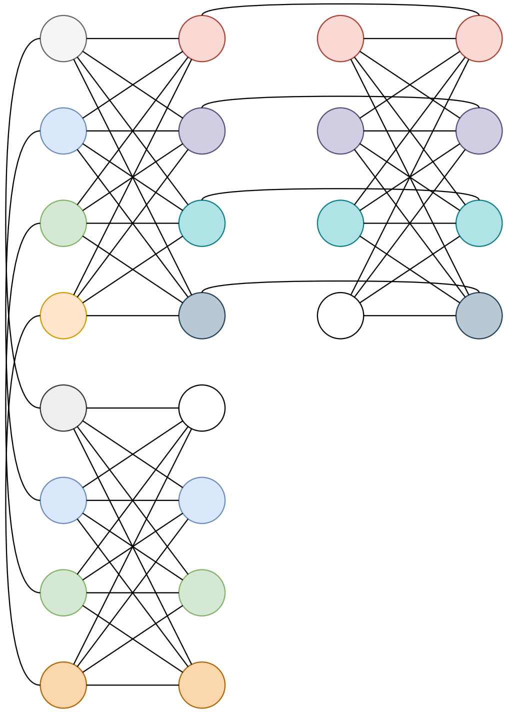


2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)





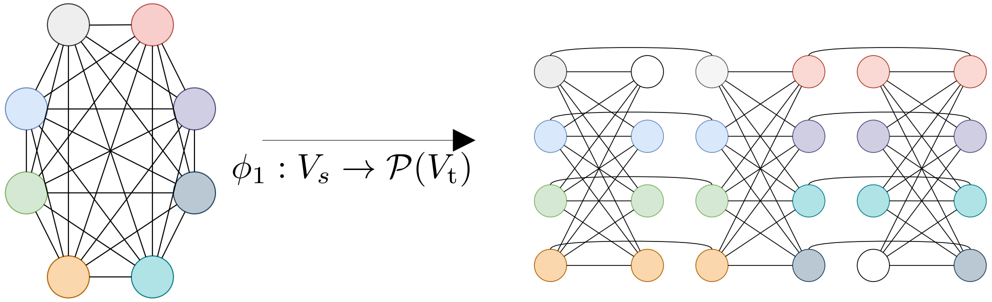


2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)

$\bar{d} = 1$

$n = 8$

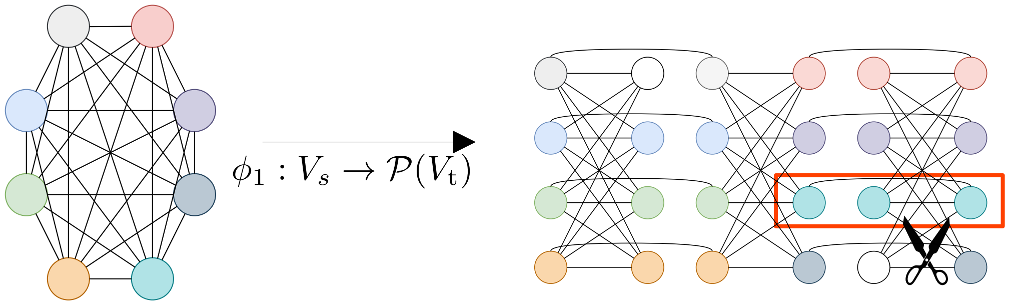


2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)

$\bar{d} = 1$

$n = 8$

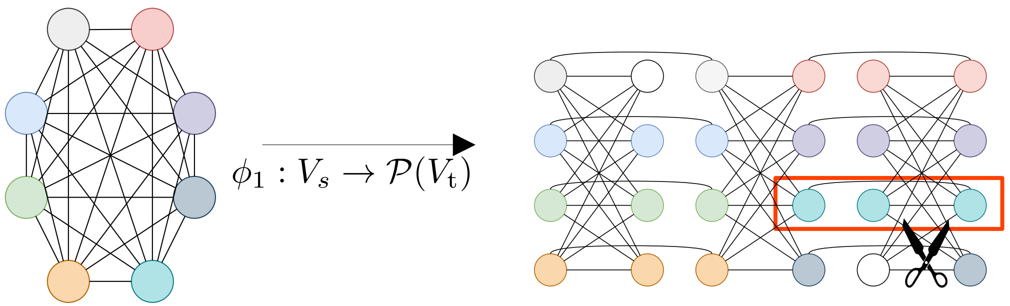


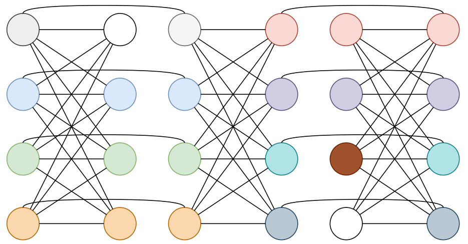
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)

$\bar{d} = 1$

$n = 8$



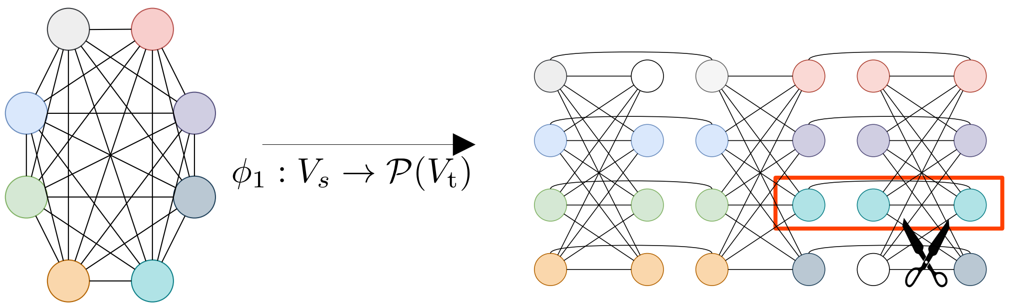


2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)

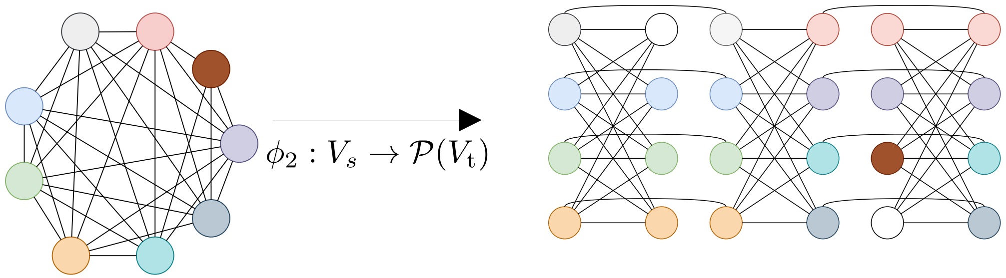
$\bar{d} = 1$

$n = 8$



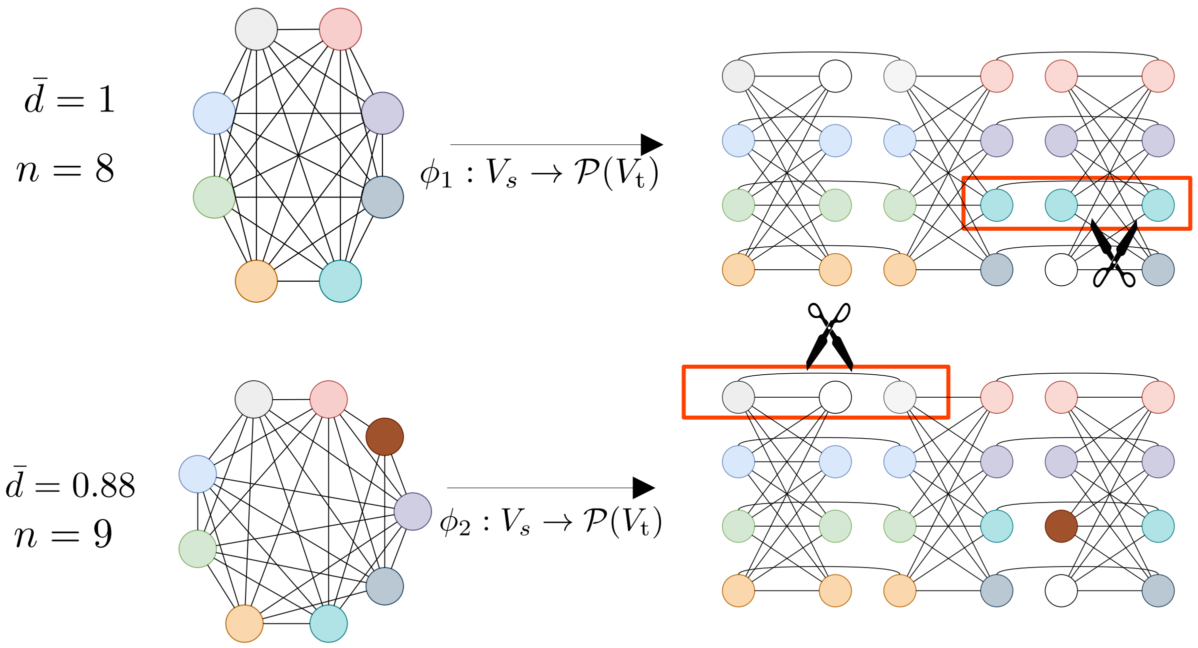
$\bar{d} = 0.88$

$n = 9$



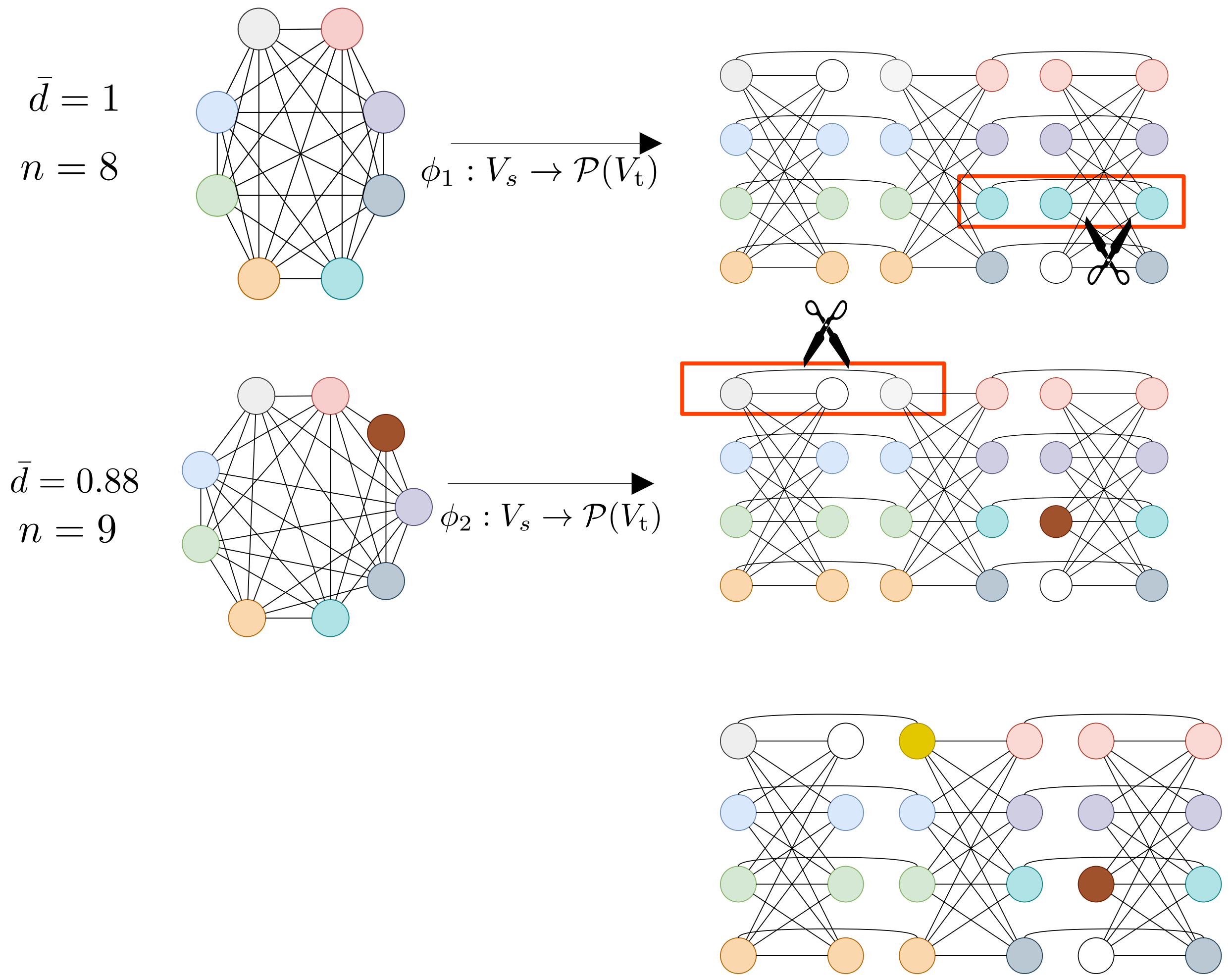
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)



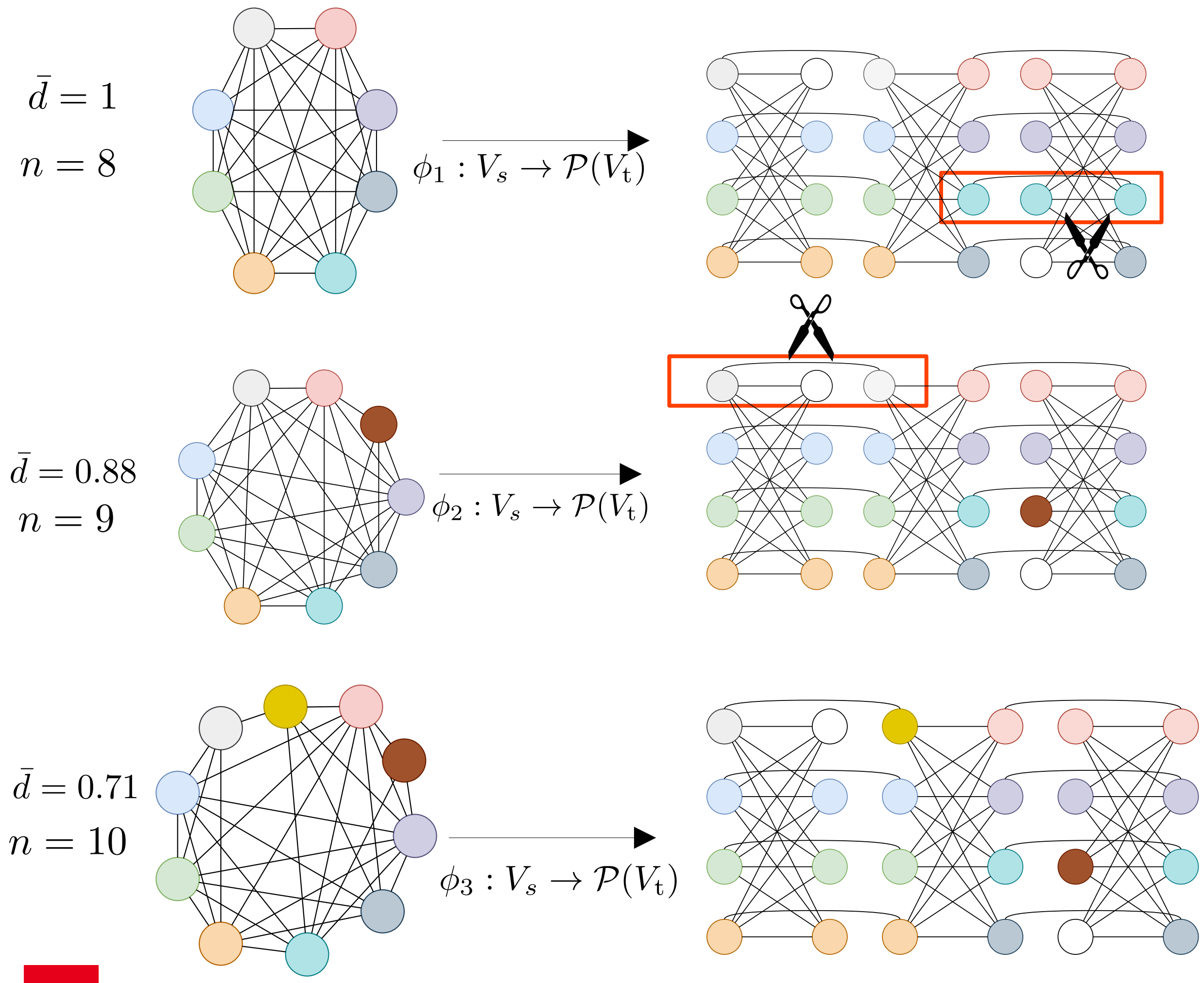
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)



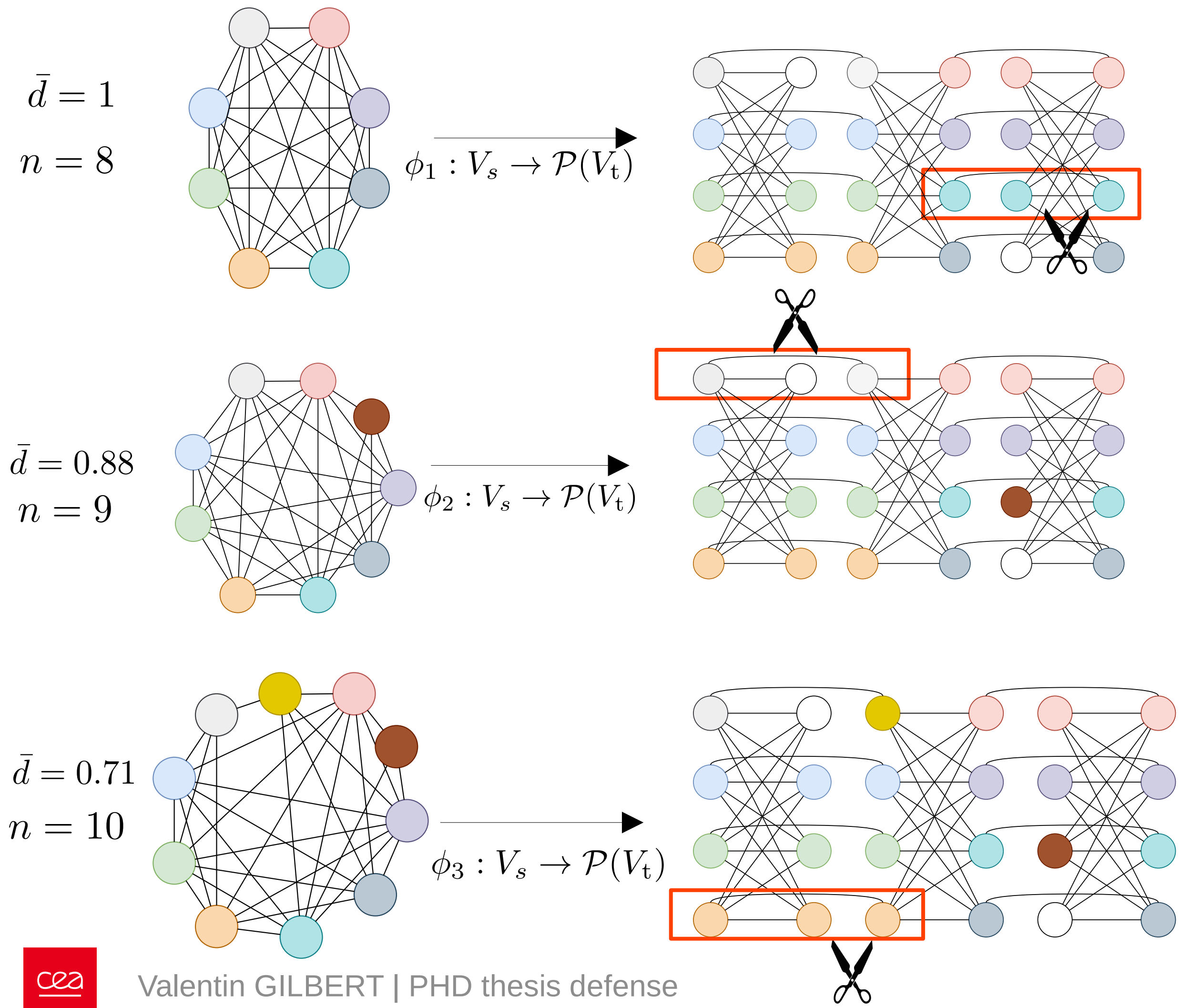
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)



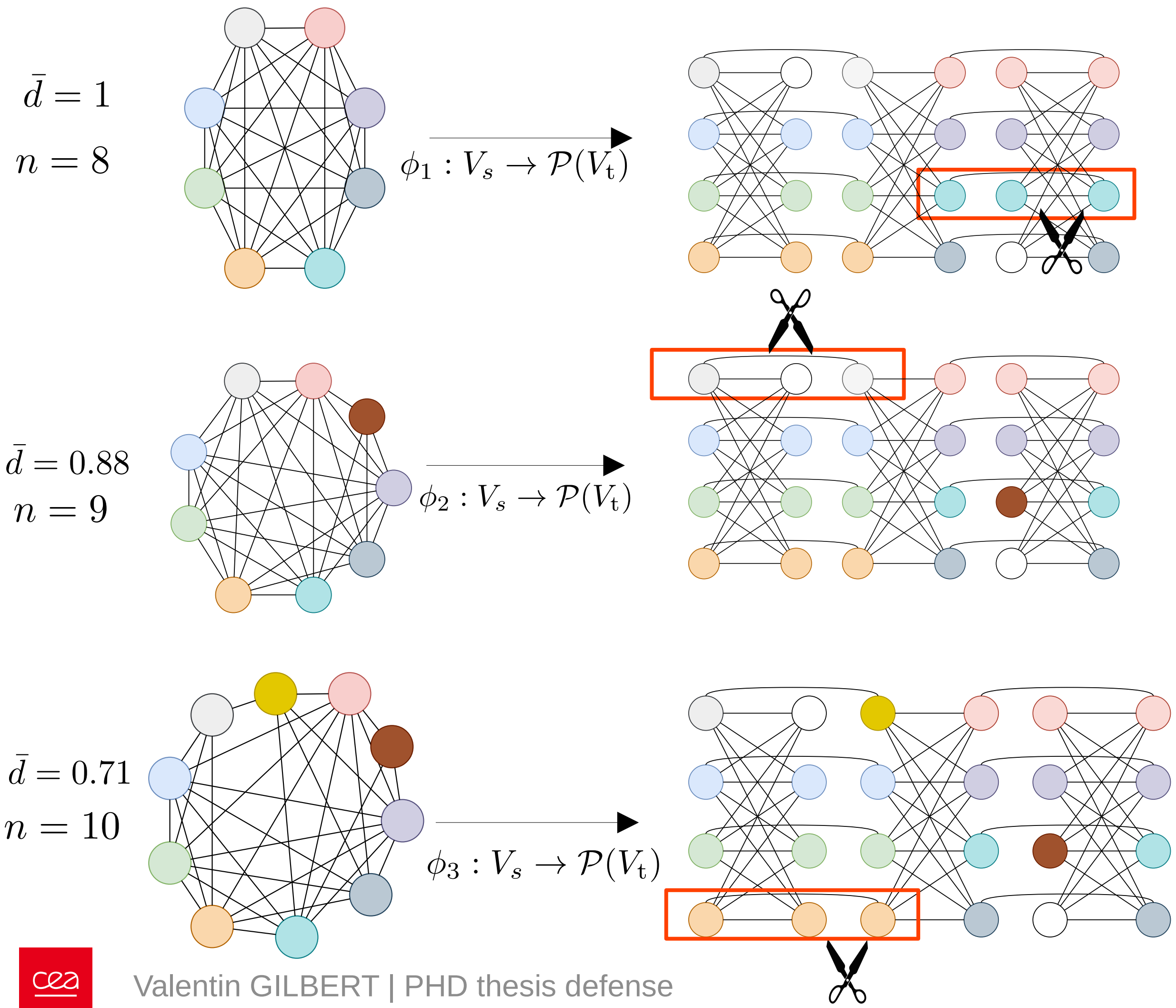
2- Generation of instances

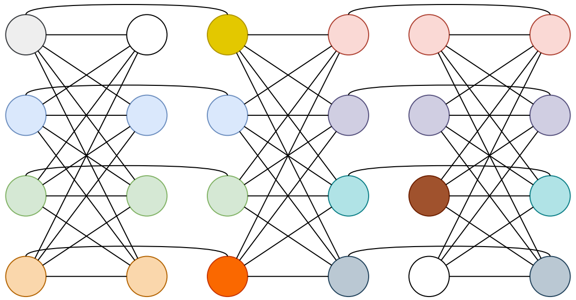
- Embedding of complete graphs (TRIAD Pattern)



2- Generation of instances

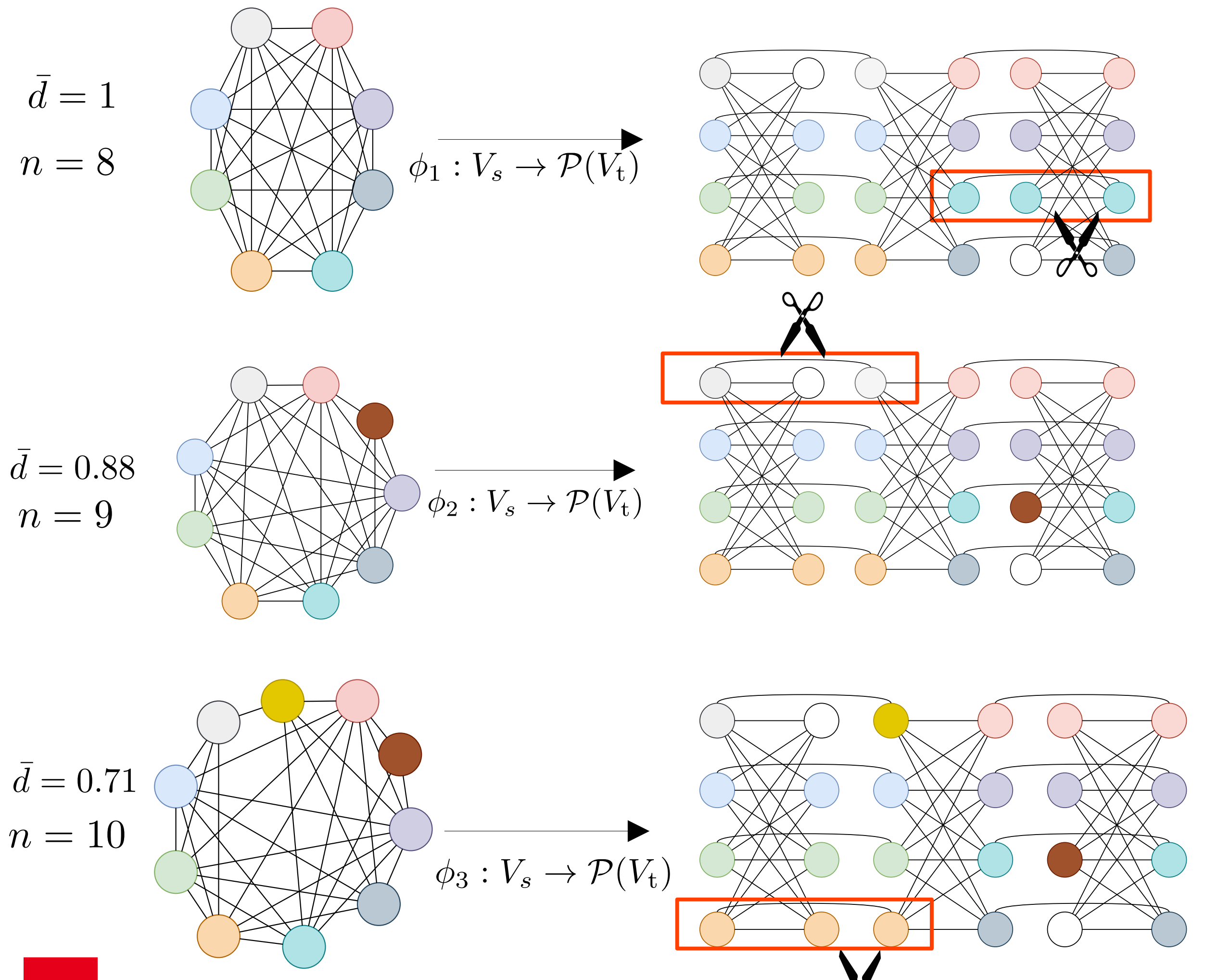
- Embedding of complete graphs (TRIAD Pattern)

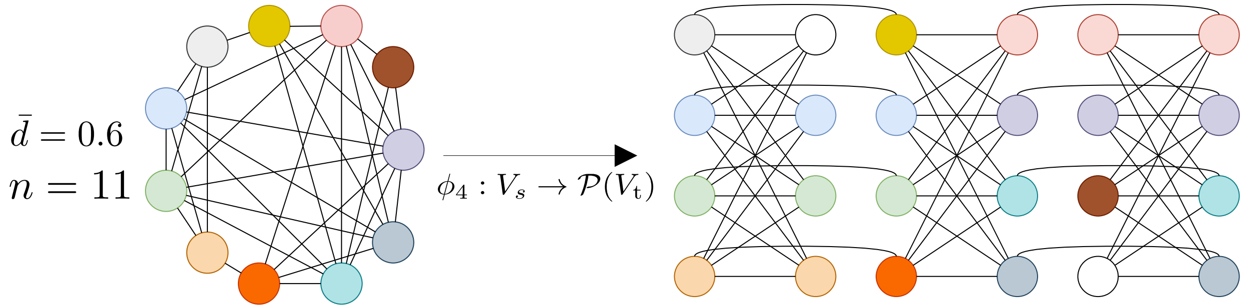




2- Generation of instances

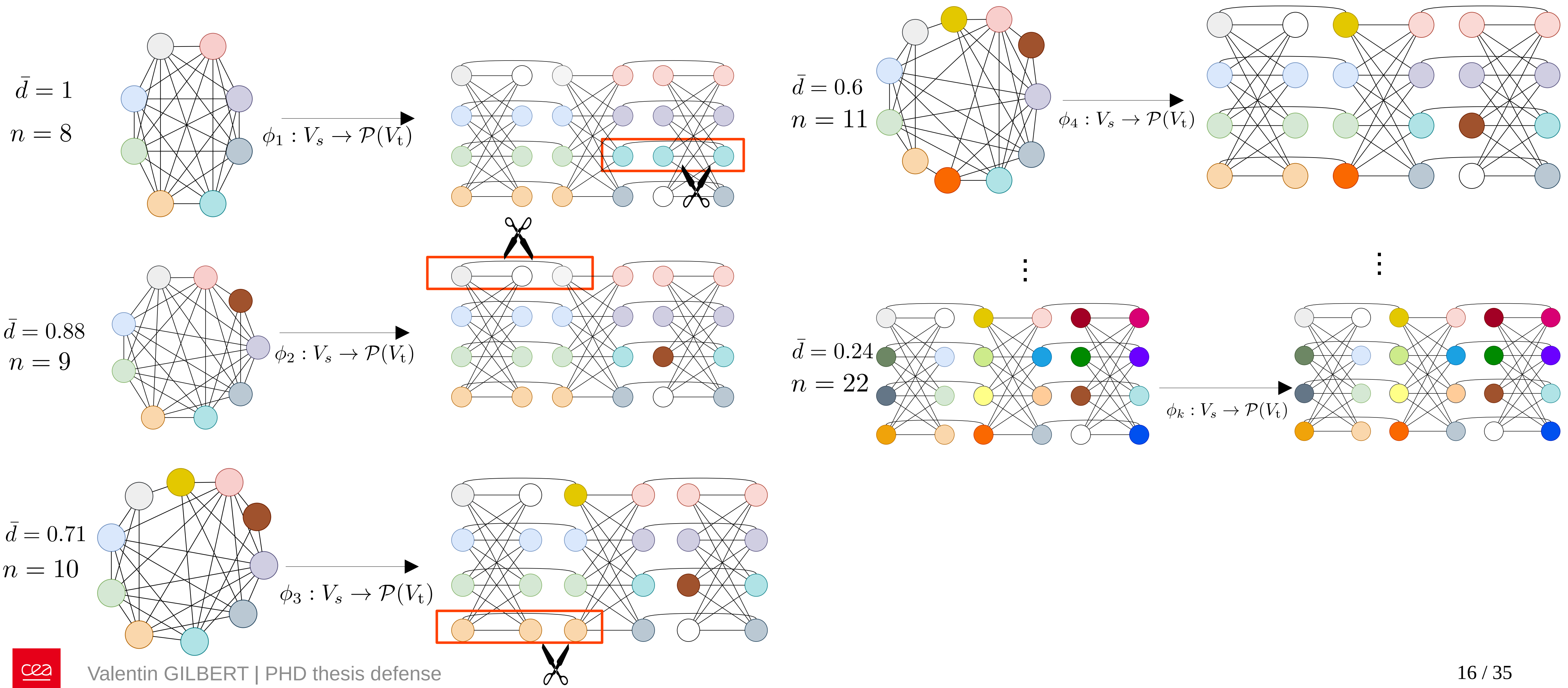
- Embedding of complete graphs (TRIAD Pattern)





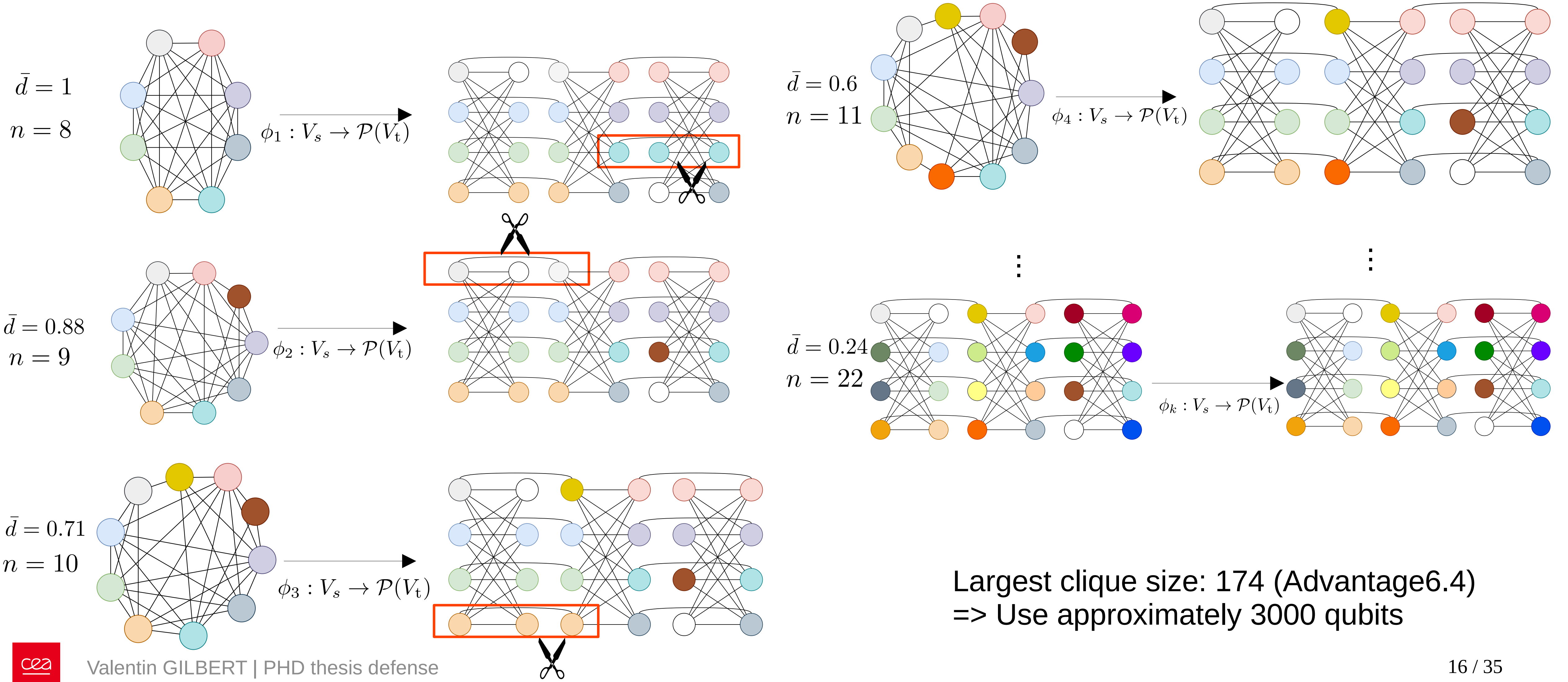
2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)



2- Generation of instances

- Embedding of complete graphs (TRIAD Pattern)



Largest clique size: 174 (Advantage6.4)
=> Use approximately 3000 qubits

2- Performance assessment

- Time To Solution metric (Gold standard) [RWJ+14]

  Number of runs:

$$R = \left\lceil \frac{\log(1-p)}{\log(1-s)} \right\rceil$$

p : probability of getting the ground state in R runs

s : Empirical success probability

$$TTS = t_a \times R$$

t_a : Time to perform a single quantum run

2- Performance assessment

- Time To Solution metric (Gold standard) [RWJ+14]

  Number of runs:

$$R = \left\lceil \frac{\log(1-p)}{\log(1-s)} \right\rceil$$

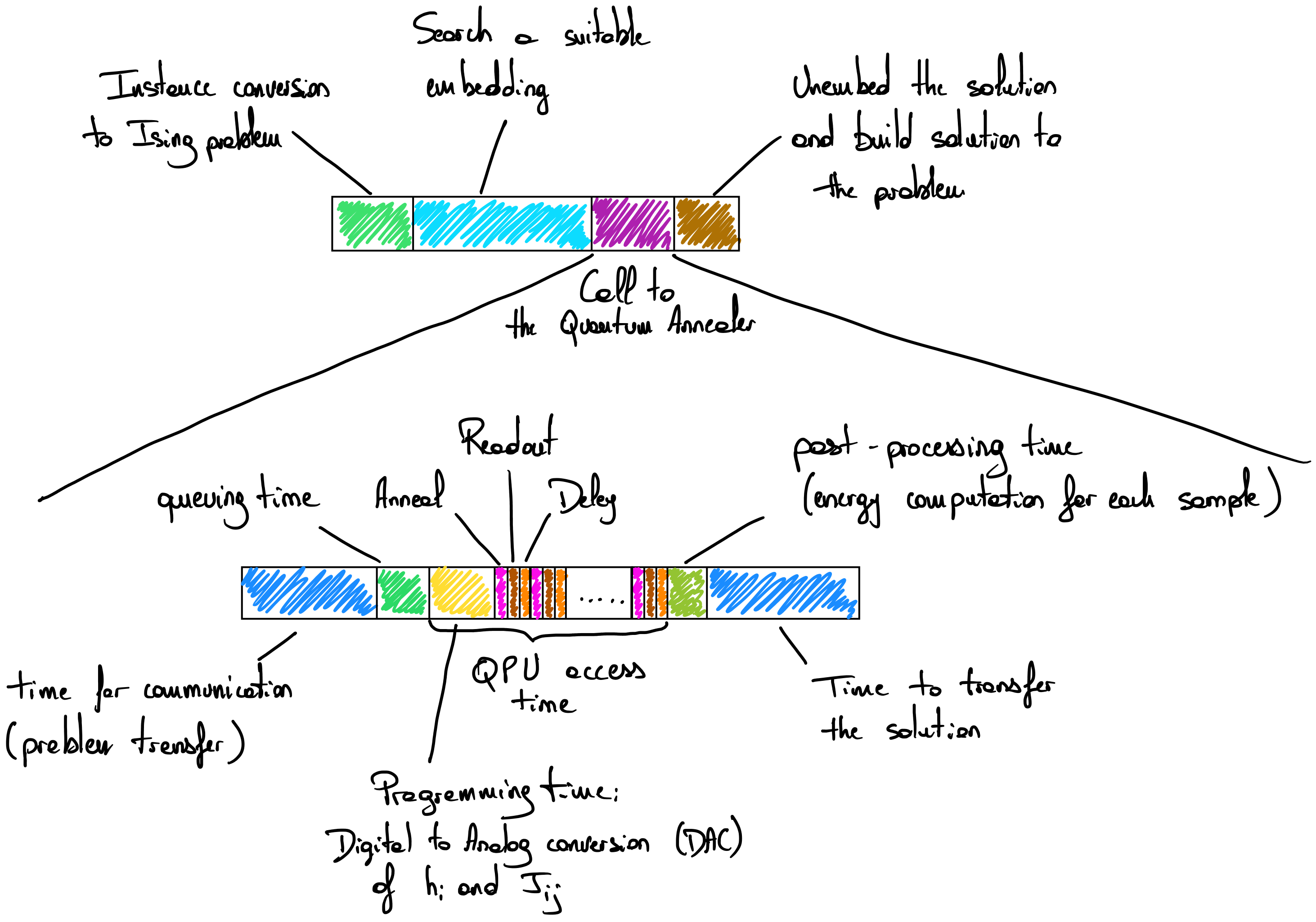
  p : probability of getting the ground state in R runs

  s : Empirical success probability

$$TTS = t_a \times R$$

  t_a : Time to perform a single quantum run

- D-Wave time repartition



2- Performance assessment

- Time To Solution metric (Gold standard) [RWJ+14]

  Number of runs:

$$R = \left\lceil \frac{\log(1-p)}{\log(1-s)} \right\rceil$$

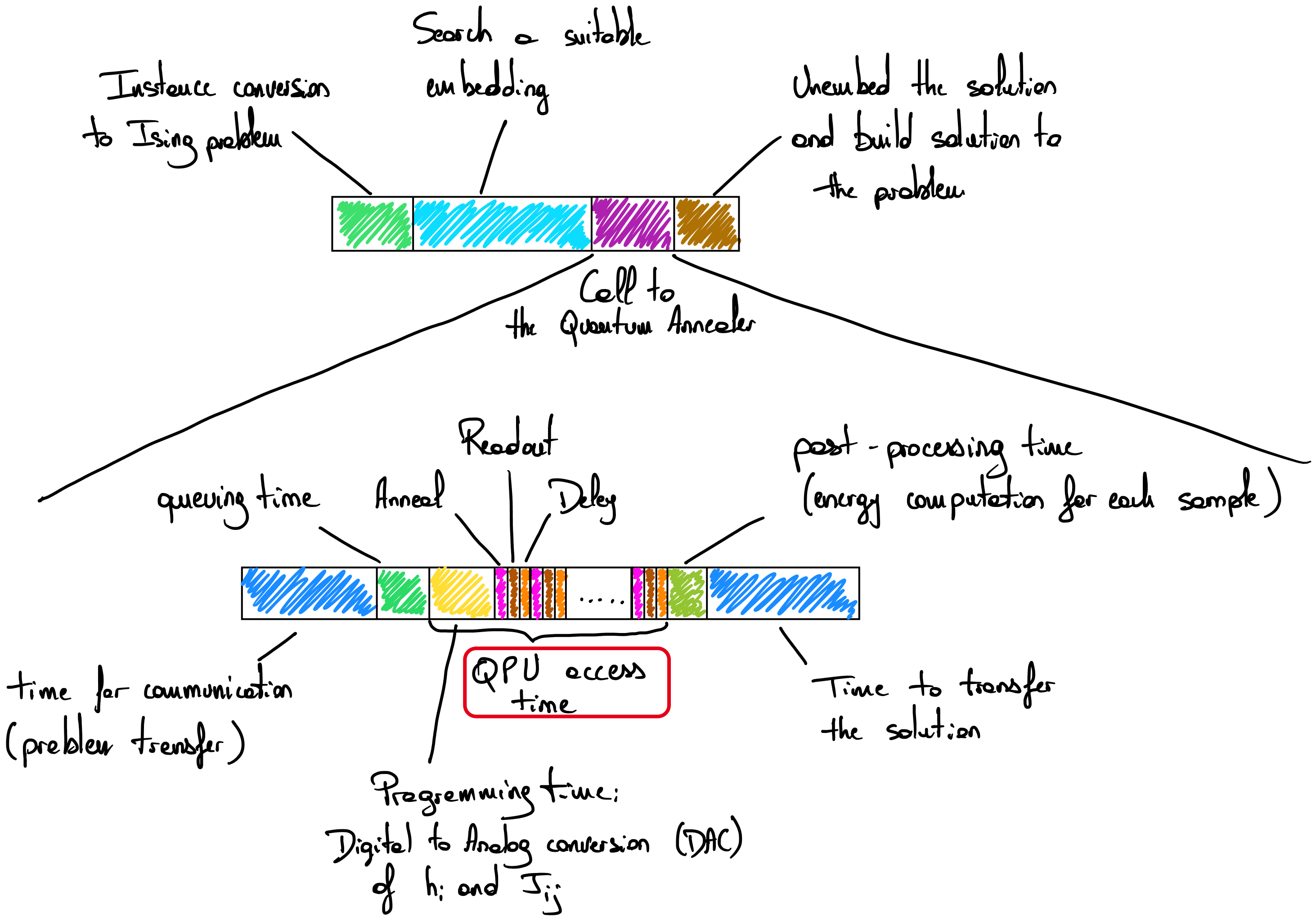
  p : probability of getting the ground state in R runs

  s : Empirical success probability

$$TTS = t_a \times R$$

  t_a : Time to perform a single quantum run

- D-Wave time repartition



2- Results – Max-cut problem

Density	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Avg $\|V_s\|$	1318	1062	912	810	737	680	635	597	565	395	321	277	248	226	209	195	184

2- Results – Max-cut problem

Density	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Avg $\|V_s\|$	1318	1062	912	810	737	680	635	597	565	395	321	277	248	226	209	195	184

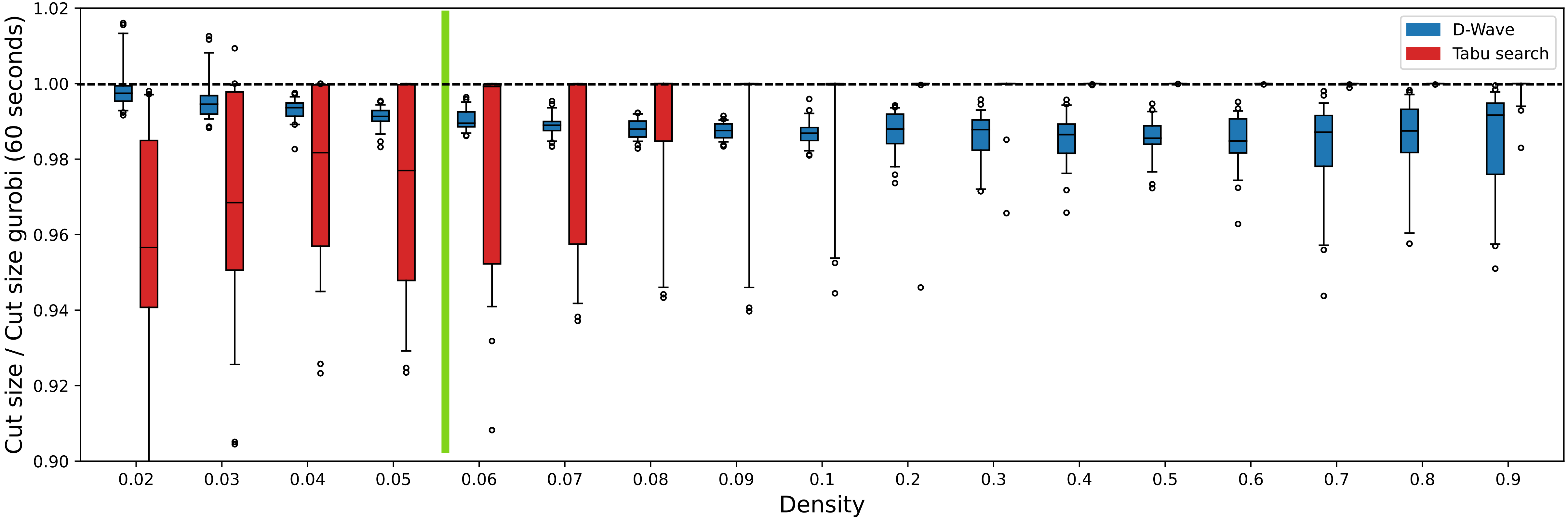
Time windows:
60s: Gurobi (reference solution)
1s: D-Wave
1s: Tabu Search

2- Results – Max-cut problem

Density	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Avg $\lvert V_s \rvert$	1318	1062	912	810	737	680	635	597	565	395	321	277	248	226	209	195	184

<u>Time windows:</u>
60s: Gurobi (reference solution)
1s: D-Wave
1s: Tabu Search

Performance intersection with Tabu Search
----- Reference solution



2- Results – MWIS problem

Density	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Avg $\|V_s\|$	1318	1062	912	810	737	680	635	597	565	395	321	277	248	226	209	195	184

2- Results – MWIS problem

Density	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Avg $\|V_s\|$	1318	1062	912	810	737	680	635	597	565	395	321	277	248	226	209	195	184

Time windows:
60s: Gurobi (reference solution)

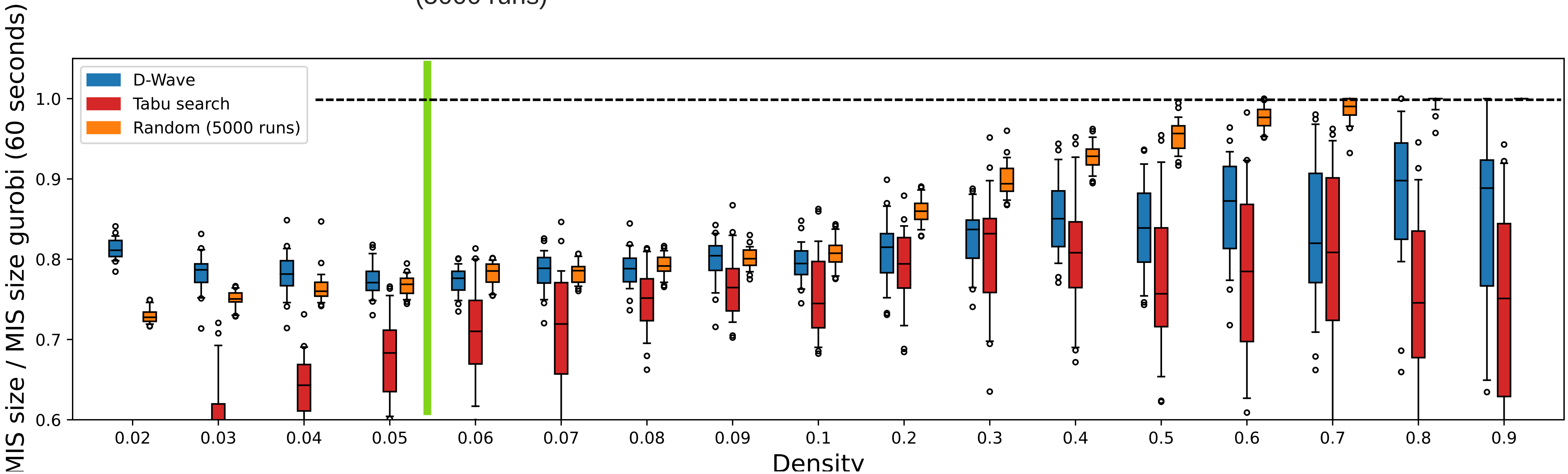
1s: D-Wave (5000 shots)
1s: Tabu Search
Random Greedy Search (5000 runs)

2- Results – MWIS problem

Density	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Avg $\|V_s\|$	1318	1062	912	810	737	680	635	597	565	395	321	277	248	226	209	195	184

Time windows:
60s: Gurobi (reference solution)
1s: D-Wave (5000 shots)
1s: Tabu Search
Random Greedy Search (5000 runs)

Performance intersection with Random Greedy search
----- Reference solution

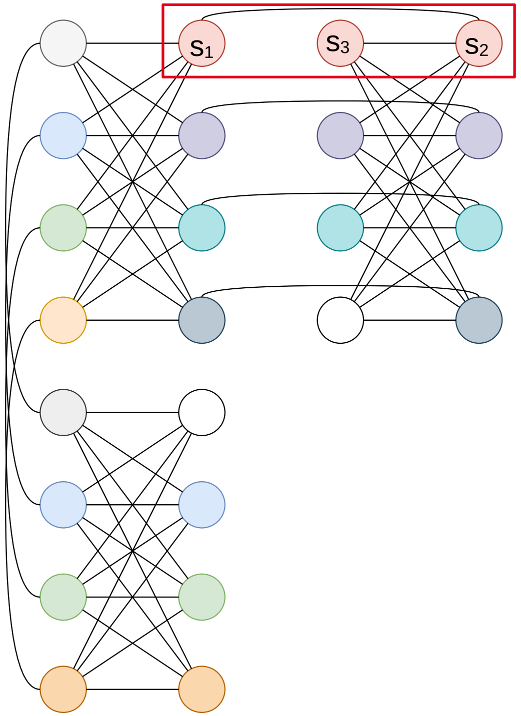


3. Contribution #2 Increasing the performance of Quantum Annealers

Generated with openai

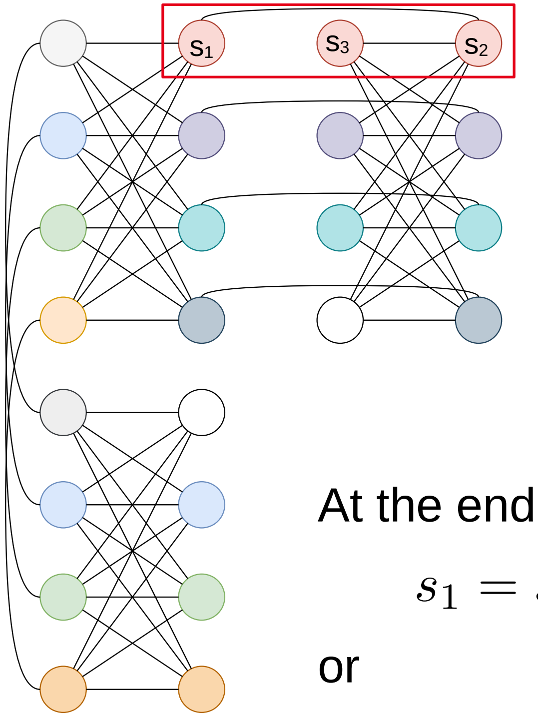
3- The embedding problem II

- Embedding step produces chains of qubits



3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

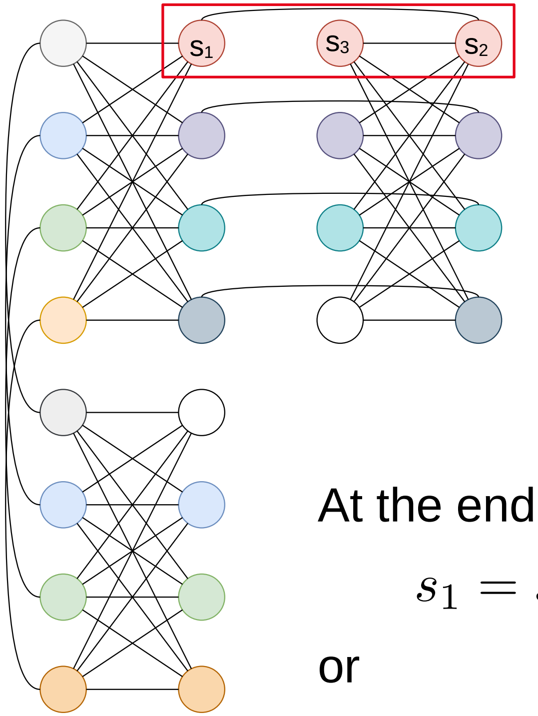
$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

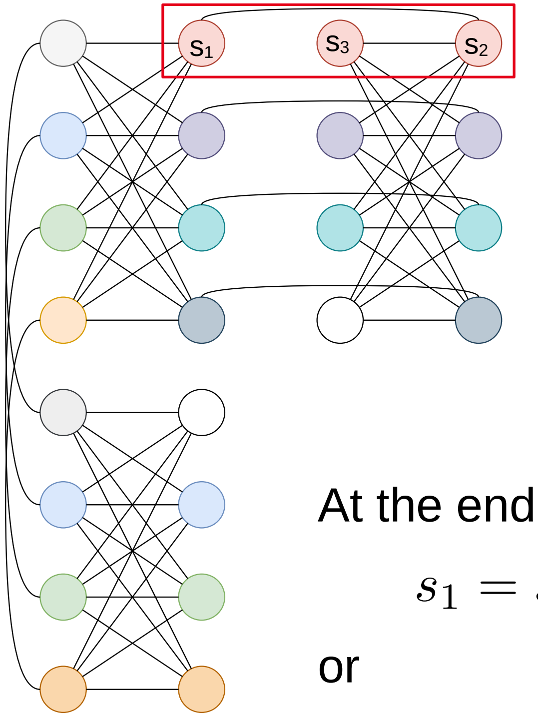
$$s_1 = s_2 = s_3 = -1$$

Let's set:

$$J_{12} = -\infty \qquad J_{23} = -\infty$$

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

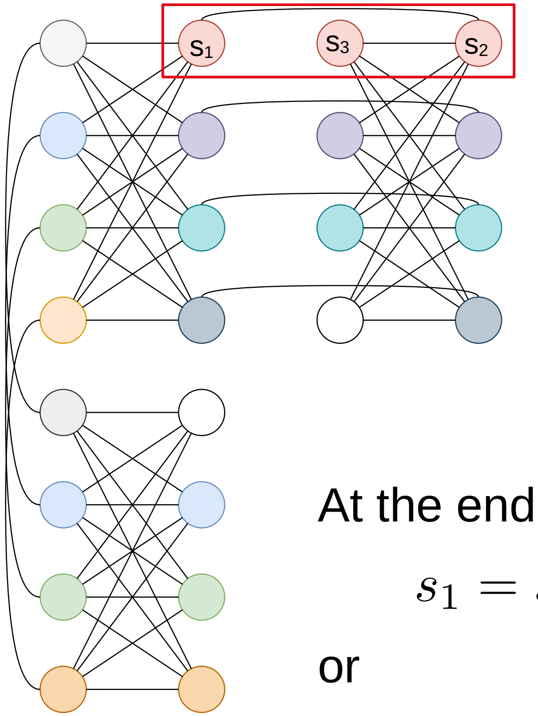
Let's set:

$$J_{12} = -\infty \qquad J_{23} = -\infty$$

Limited programming range

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

Let's set:

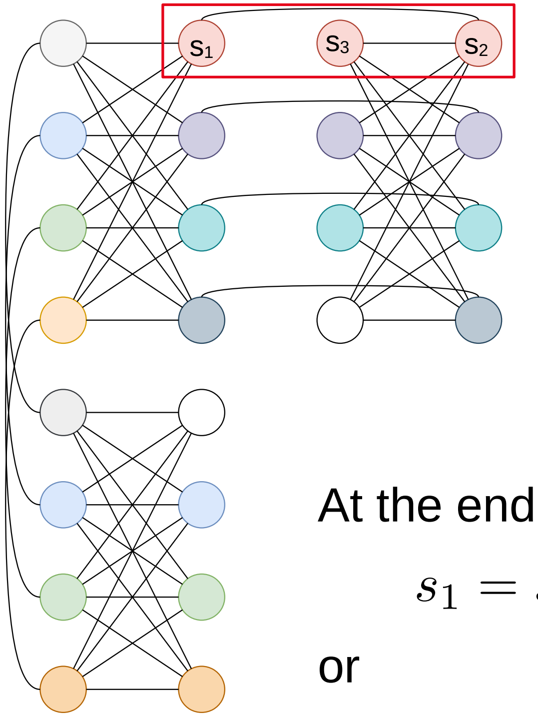
$$J_{12} = -\infty \qquad J_{23} = -\infty$$

Limited programming range

- How to set the chain between these qubits ?

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

Let's set:

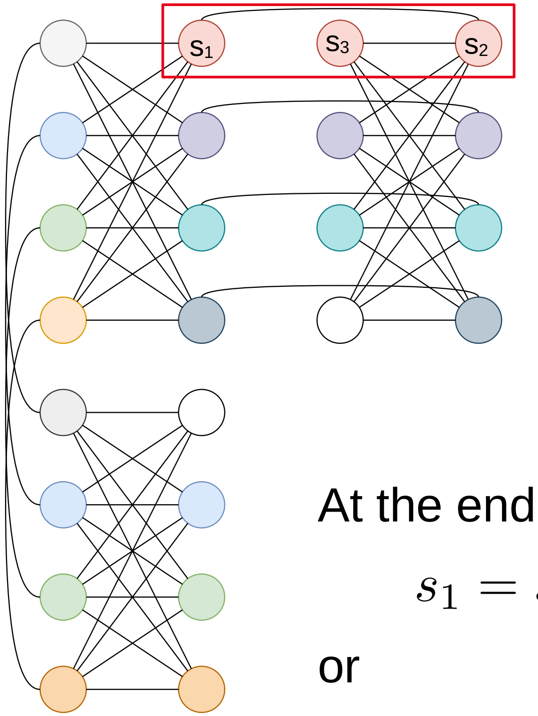
$$J_{12} = -\infty \qquad J_{23} = -\infty$$

Limited programming range

- How to set the chain between these qubits ?
  - Average problem setting [VMK+14]

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

Let's set:

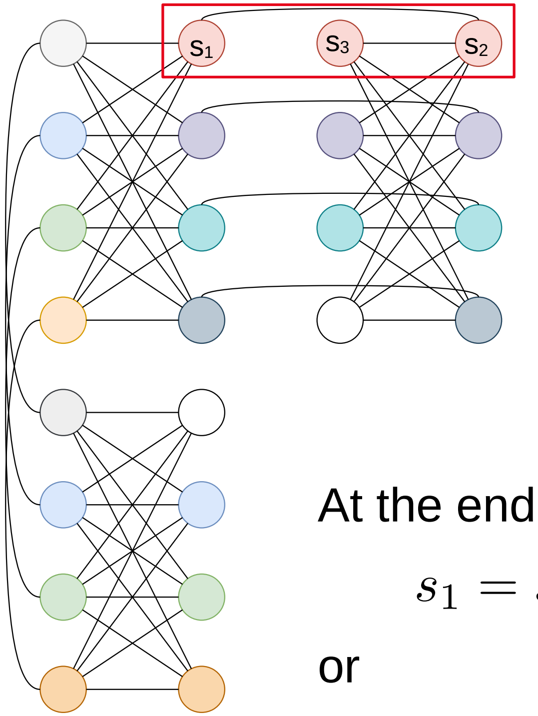
$$J_{12} = -\infty \qquad J_{23} = -\infty$$

Limited programming range

- How to set the chain between these qubits ?
  - Average problem setting [VMK+14]
  - Upper bonds [Cho08]

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

Let's set:

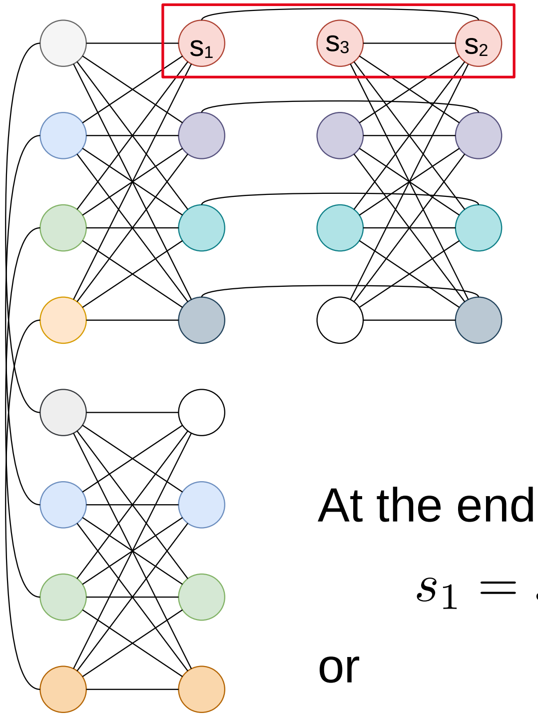
$$J_{12} = -\infty \qquad J_{23} = -\infty$$

Limited programming range

- How to set the chain between these qubits ?
  - Average problem setting [VMK+14]
  - Upper bonds [Cho08]
  - Basic Scan [HIM+18] [WWC+22]

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

Let's set:

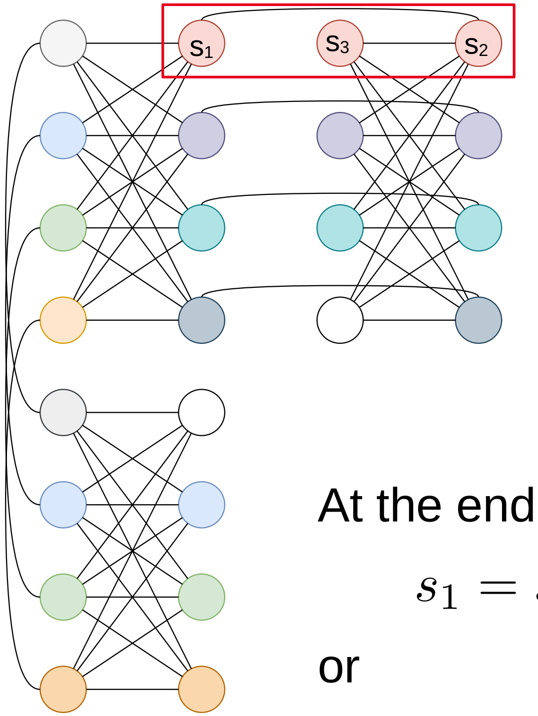
$$J_{12} = -\infty \qquad J_{23} = -\infty$$

Limited programming range

- How to set the chain between these qubits ?
  - Average problem setting [VMK+14]
  - Upper bonds [Cho08]
  - Basic Scan [HIM+18] [WWC+22]
  - Advanced algorithms [Dji23]

3- The embedding problem II

- Embedding step produces chains of qubits



At the end of the annealing:

$$s_1 = s_2 = s_3 = +1$$

or

$$s_1 = s_2 = s_3 = -1$$

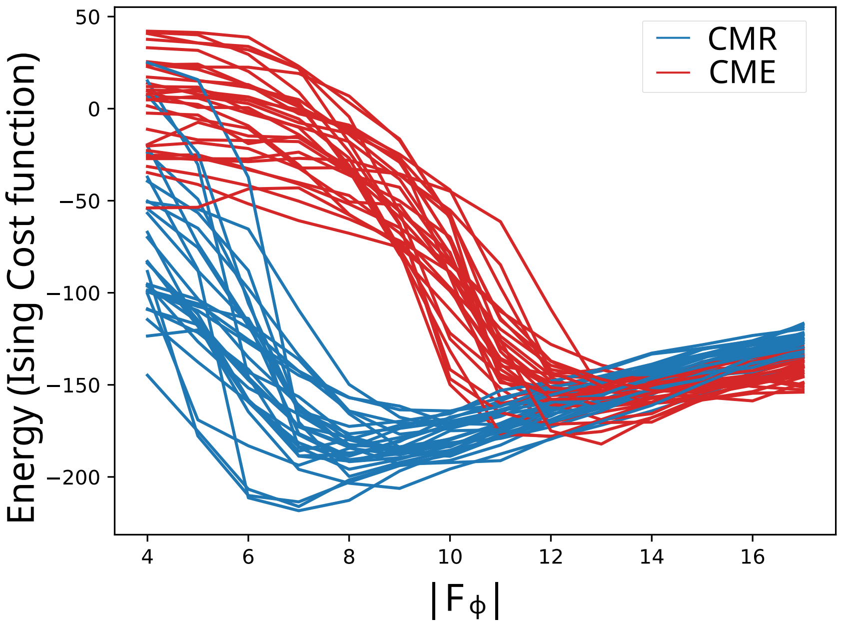
Let's set:

$$J_{12} = -\infty \qquad J_{23} = -\infty$$

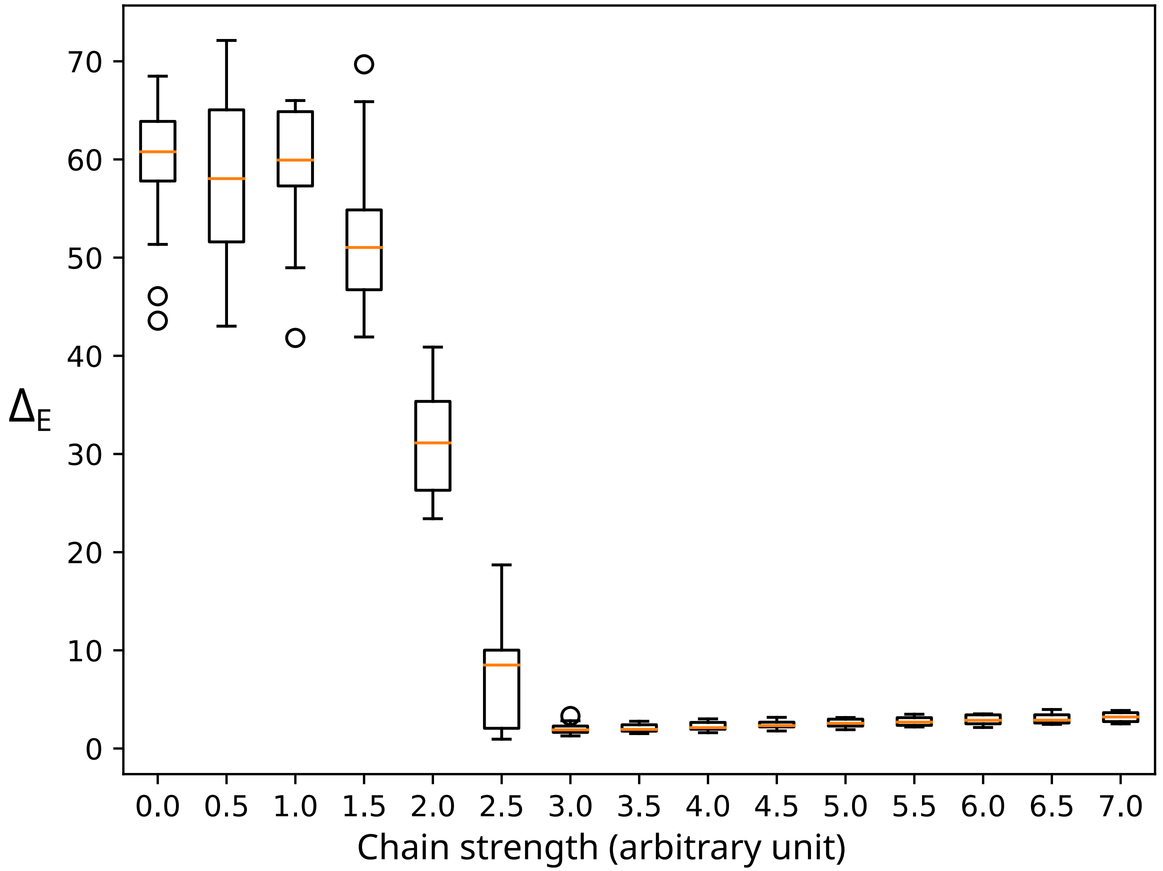
⚠ Limited programming range

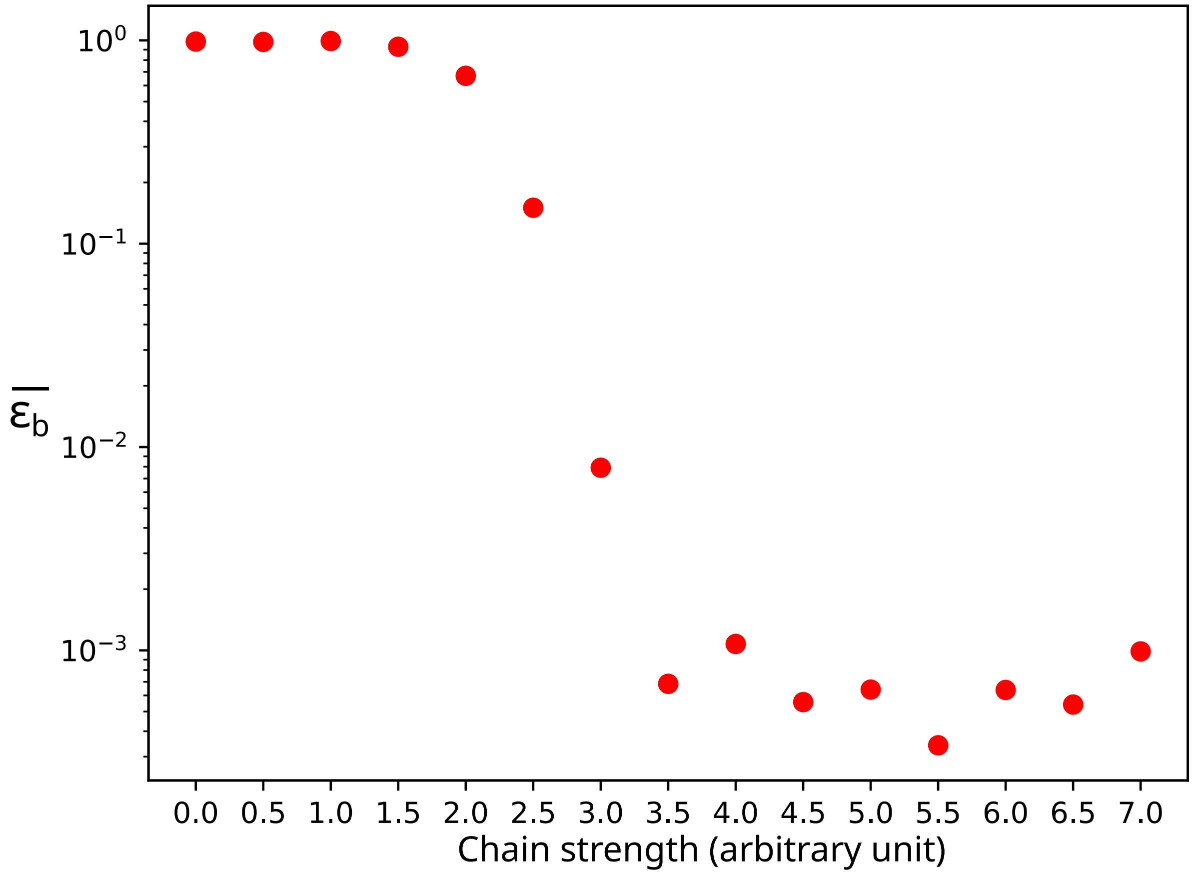
- How to set the chain between these qubits ?
  - Average problem setting [VMK+14]
  - Upper bonds [Cho08]
  - Basic Scan [HIM+18] [WWC+22]
  - Advanced algorithms [Dji23]

- Global chain strength: $F_\phi < 0$

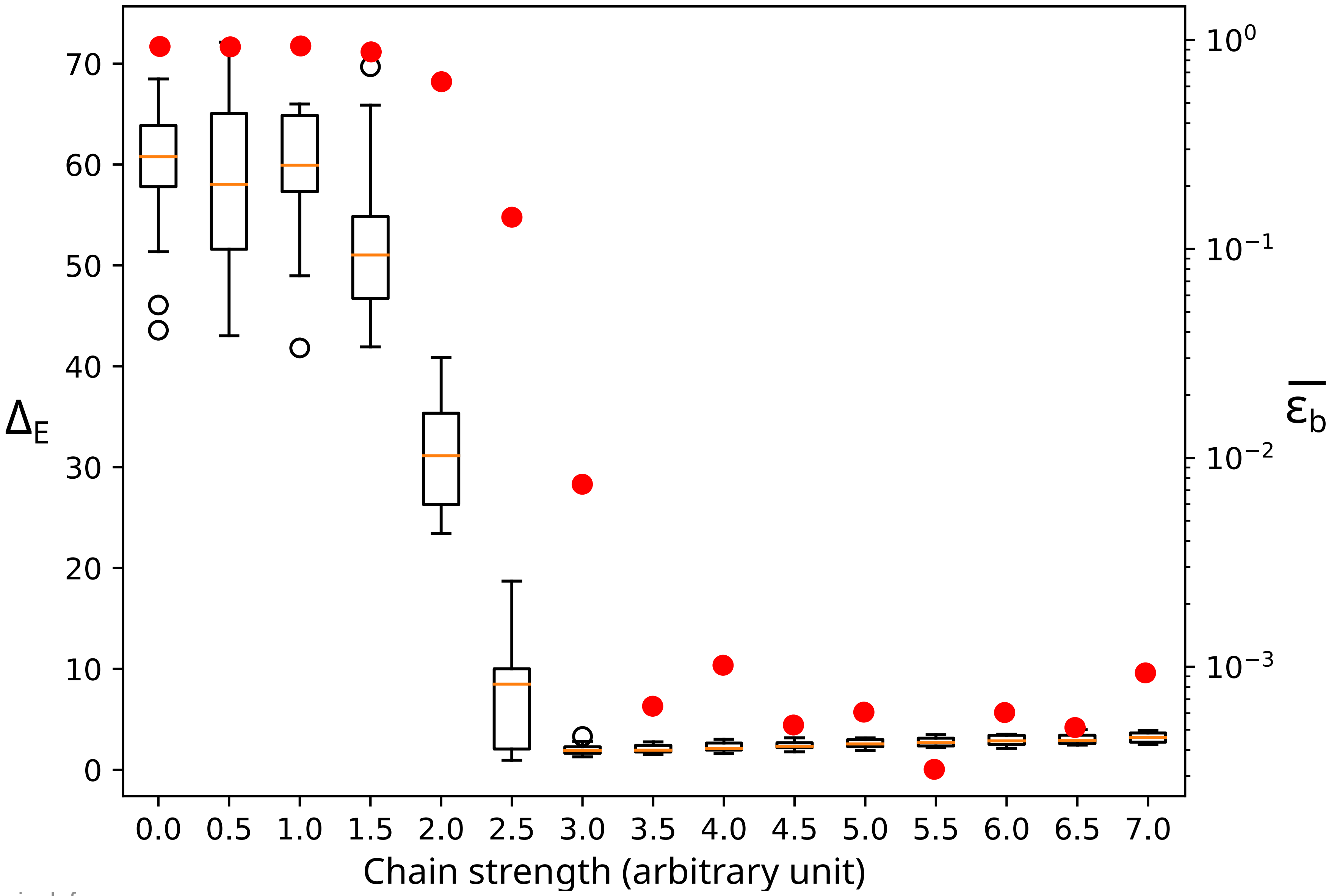


3- Chain scan & Chain breaks

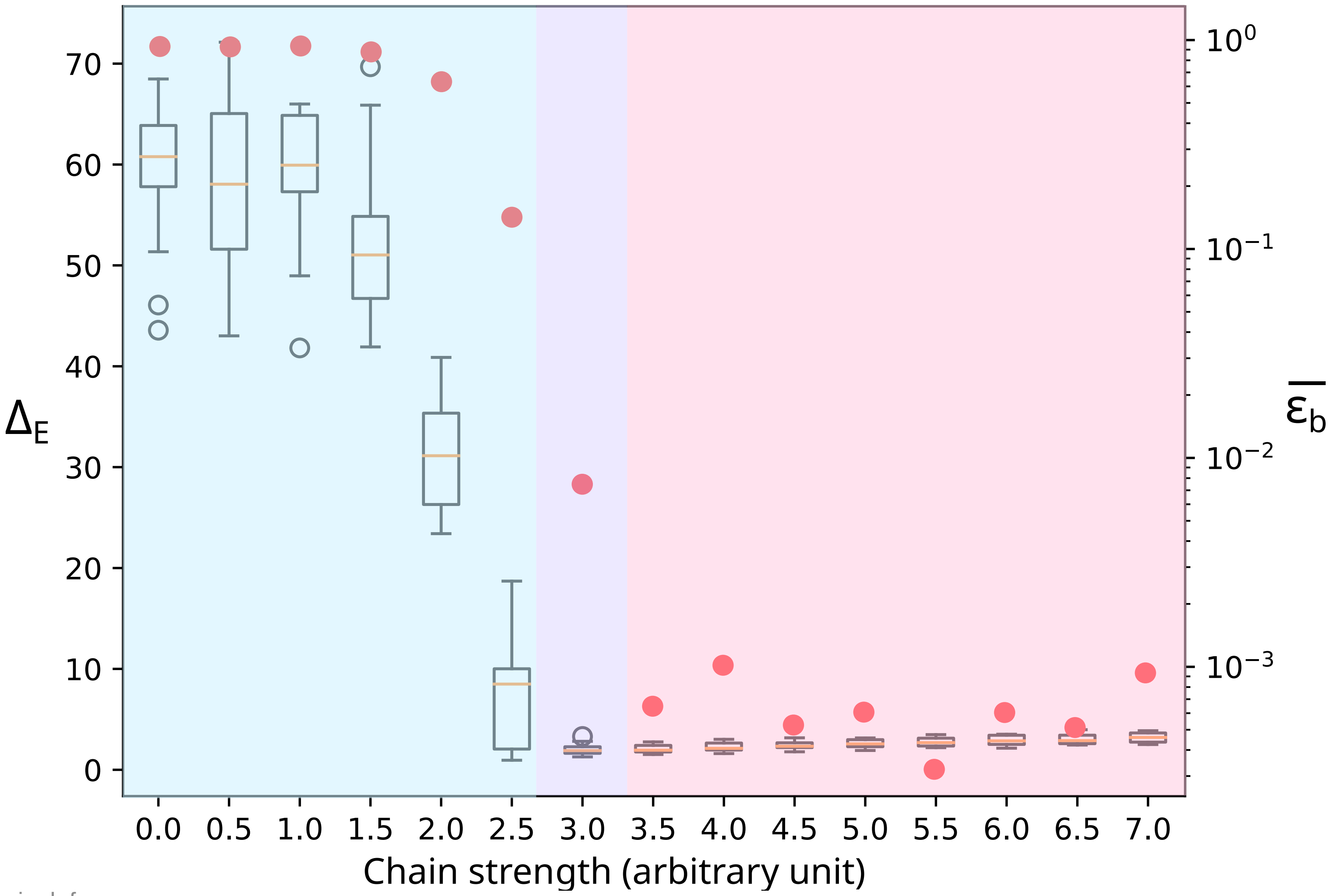




3- Chain scan & Chain breaks



3- Chain scan & Chain breaks



3- Chain scan & Chain breaks

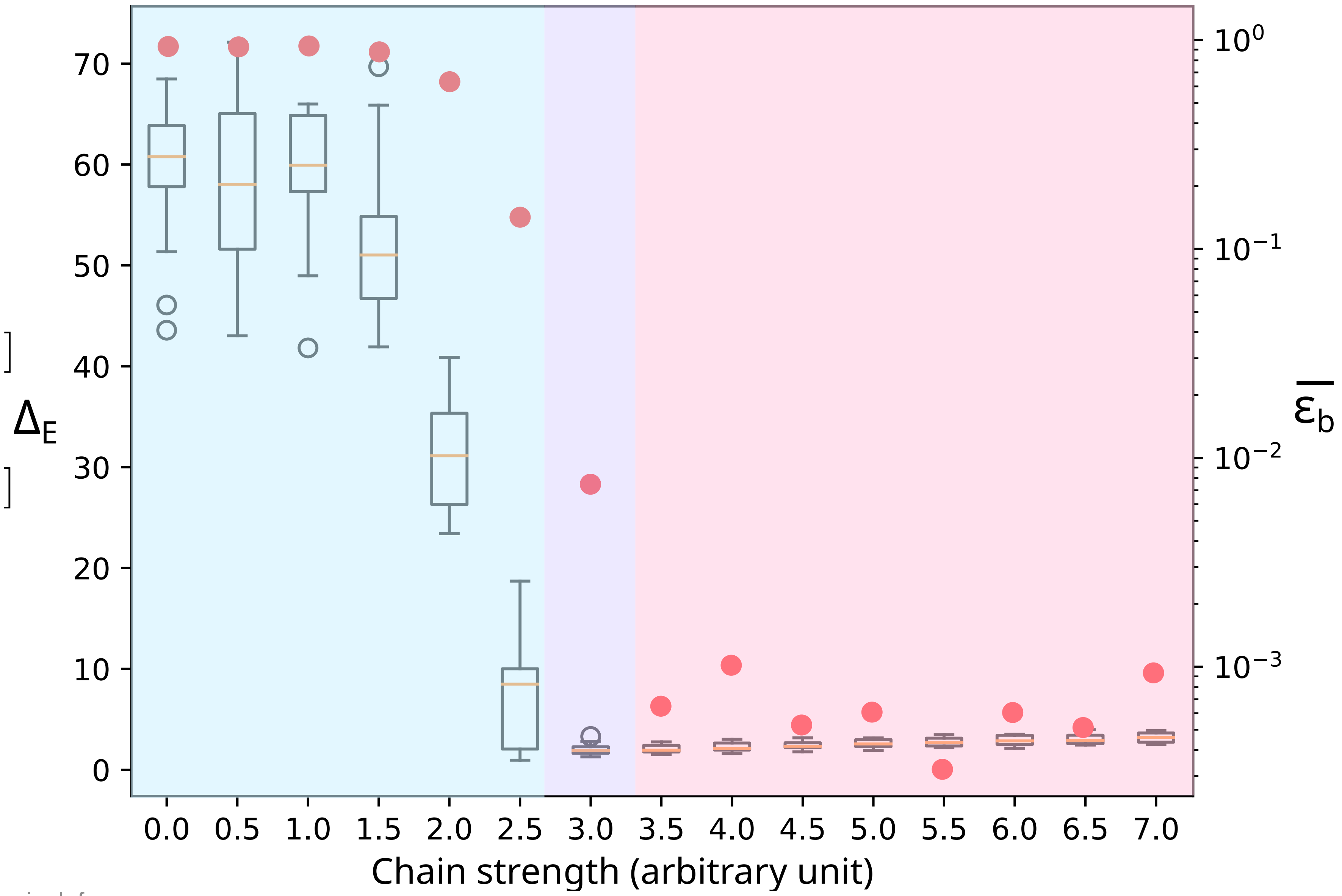
Chain break intervals:

Advantage2:

$[6 \times 10^{-3}, 2 \times 10^{-2}]$

Advantage6.4:

$[2 \times 10^{-2}, 5 \times 10^{-2}]$



3- Results

- 30 instances of unweighted max-cut for each density
- Shot / pre-processing step: 128
- Final run number of shots:

  Advantage2: 3072

  Advantage6.4: 4096

Advantage2_prototype2.2			Best cut size	Cut size improvement				Step
Instance size	Density	Embedding		min	max	mean	std	
$n = 40$	0.1	CMR	66.4	+0%	+0%	+0%	0%	5.4
	0.5	CMR	243	+0%	+2%	+0.2%	0%	5.8
	0.9	CMR	362.8	+2.1%	+8.2%	+5%	1.6%	4.6
$n = 80$	0.1	CMR	235.5	+0%	+0%	+0%	0%	4.7
	0.5	CME	804	+9.8%	+17.2%	+12.5%	0.2%	4.2
	0.9	CME	1435	+2%	+4.7%	+3.2%	0.6%	4.2
Advantage6.4			Best Cut size	Cut size improvement				Step
Instance size	Density	Embedding		min	max	mean	std	
$n = 100$	0.1	CMR	355.9	+0%	+0.3%	+0%	0%	4.5
	0.5	CME	1271.4	+5.6%	+14.5%	+8.8%	1.8%	2.7
	0.9	CME	2243	+1.4%	+3.7%	+2.5%	0.5%	3.7
$n = 170$	0.1	CMR	950.8	−2.1%	+0.6%	−0.5%	0.5%	2.1
	0.5	CME	3631.4	+2.8%	+6.2%	+4.5%	0.7%	2.1
	0.9	CME	6519.4	+0.4%	+1.4%	+0.8%	0.2%	3.2

3- Results

- 30 instances of unweighted max-cut for each density
- Shot / pre-processing step: 128
- Final run number of shots:
  Advantage2: 3072
  Advantage6.4: 4096

Advantage2_prototype2.2			Best cut size	Cut size improvement				Step
Instance size	Density	Embedding		min	max	mean	std	
$n = 40$	0.1	CMR	66.4	+0%	+0%	+0%	0%	5.4
	0.5	CMR	243	+0%	+2%	+0.2%	0%	5.8
	0.9	CMR	362.8	+2.1%	+8.2%	+5%	1.6%	4.6
$n = 80$	0.1	CMR	235.5	+0%	+0%	+0%	0%	4.7
	0.5	CME	804	+9.8%	+17.2%	+12.5%	0.2%	4.2
	0.9	CME	1435	+2%	+4.7%	+3.2%	0.6%	4.2
Advantage6.4			Best Cut size	Cut size improvement				Step
Instance size	Density	Embedding		min	max	mean	std	
$n = 100$	0.1	CMR	355.9	+0%	+0.3%	+0%	0%	4.5
	0.5	CME	1271.4	+5.6%	+14.5%	+8.8%	1.8%	2.7
	0.9	CME	2243	+1.4%	+3.7%	+2.5%	0.5%	3.7
$n = 170$	0.1	CMR	950.8	−2.1%	+0.6%	−0.5%	0.5%	2.1
	0.5	CME	3631.4	+2.8%	+6.2%	+4.5%	0.7%	2.1
	0.9	CME	6519.4	+0.4%	+1.4%	+0.8%	0.2%	3.2

Conclusion

- Methodology to benchmark Quantum Annealers

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**
  - D-Wave Quantum Annealers are mostly adapted for **very sparse** and **unconstrained** problems

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**
  - D-Wave Quantum Annealers are mostly adapted for **very sparse** and **unconstrained** problems
  - **Fairness** evaluation of benchmarking methods:

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**
  - D-Wave Quantum Annealers are mostly adapted for **very sparse** and **unconstrained** problems
  - **Fairness** evaluation of benchmarking methods:
    - Fairness of instances set (diversity) [GLS23]

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**
  - D-Wave Quantum Annealers are mostly adapted for **very sparse** and **unconstrained** problems
  - **Fairness** evaluation of benchmarking methods:
    - Fairness of instances set (diversity) [GLS23]
    - Fairness of the benchmarking protocol (quantumbenchmarkzoo.org)

Quantum Benchmark Zoo

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**
  - D-Wave Quantum Annealers are mostly adapted for **very sparse** and **unconstrained** problems
  - **Fairness** evaluation of benchmarking methods:
    - Fairness of instances set (diversity) [GLS23]
    - Fairness of the benchmarking protocol (quantumbenchmarkzoo.org)
- Improvements of Quantum Annealers' parameter setting

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**
  - D-Wave Quantum Annealers are mostly adapted for **very sparse** and **unconstrained** problems
  - **Fairness** evaluation of benchmarking methods:
    - Fairness of instances set (diversity) [GLS23]
    - Fairness of the benchmarking protocol (quantumbenchmarkzoo.org)

Quantum Benchmark Zoo

- Improvements of Quantum Annealers' parameter setting
  - The parameters list is long and complex (interdependence between parameters)

Conclusion

- Methodology to benchmark Quantum Annealers
  - Generation of Instances of **gradual difficulty**
  - D-Wave Quantum Annealers are mostly adapted for **very sparse** and **unconstrained** problems
  - **Fairness** evaluation of benchmarking methods:
    - Fairness of instances set (diversity) [GLS23]
    - Fairness of the benchmarking protocol (quantumbenchmarkzoo.org)

Quantum Benchmark Zoo

- Improvements of Quantum Annealers' parameter setting
  - The parameters list is long and complex (interdependence between parameters)
  - The optimization of these parameters should be included in the TTS metric

Perspectives

- Benchmarking

Perspectives

- Benchmarking
  - Possible extension of our approach to benchmark the QAOA (Optimal planted swapping network)

Perspectives

- Benchmarking
  - Possible extension of our approach to benchmark the QAOA (Optimal planted swapping network)
  - En route to the Algorithm Selection Problem [MCD20] and Instance Space Analysis [SM23]?

Perspectives

- Benchmarking
  - Possible extension of our approach to benchmark the QAOA (Optimal planted swapping network)
  - En route to the Algorithm Selection Problem [MCD20] and Instance Space Analysis [SM23]?
  - Protocols to evaluate the fairness of benchmarking experiments

Perspectives

- Benchmarking
  - Possible extension of our approach to benchmark the QAOA (Optimal planted swapping network)
  - En route to the Algorithm Selection Problem [MCD20] and Instance Space Analysis [SM23]?
  - Protocols to evaluate the fairness of benchmarking experiments

- Quantum Annealer parameters

Perspectives

- Benchmarking
  - Possible extension of our approach to benchmark the QAOA (Optimal planted swapping network)
  - En route to the Algorithm Selection Problem [MCD20] and Instance Space Analysis [SM23]?
  - Protocols to evaluate the fairness of benchmarking experiments

- Quantum Annealer parameters
  - Interdependence of parameters (annealing schedules, annealing time, chain strength etc..)

    => Use of holistic method to tune these parameters all together?

Perspectives

- Benchmarking
  - Possible extension of our approach to benchmark the QAOA (Optimal planted swapping network)
  - En route to the Algorithm Selection Problem [MCD20] and Instance Space Analysis [SM23]?
  - Protocols to evaluate the fairness of benchmarking experiments

- Quantum Annealer parameters
  - Interdependence of parameters (annealing schedules, annealing time, chain strength etc..)

    => Use of holistic method to tune these parameters all together?
  - Recent access to very short annealing times and quenched schedules (simulation of closed system dynamics)

    => better understanding of the dynamics of quantum systems?

Perspectives

- Benchmarking
  - Possible extension of our approach to benchmark the QAOA (Optimal planted swapping network)
  - En route to the Algorithm Selection Problem [MCD20] and Instance Space Analysis [SM23]?
  - Protocols to evaluate the fairness of benchmarking experiments

- Quantum Annealer parameters
  - Interdependence of parameters (annealing schedules, annealing time, chain strength etc..)

    => Use of holistic method to tune these parameters all together?
  - Recent access to very short annealing times and quenched schedules (simulation of closed system dynamics)

    => better understanding of the dynamics of quantum systems?

> “*NISQ will not change the world by itself, at least not right away; instead we should regard it as a step toward more powerful quantum technologies we hope to develop in the future.*”
> J. Preskill [Pre21]

Thank you !

A big thank to ACTIF (Association CEA des thésard.e.s d'île-de-France)

New president: Lise.jolicoeur@cea.fr

Publications

G. Bettonte, V. Gilbert, D. Vert, S. Louise, and R. Sirdey, “Quantum approaches for wcet-related optimization problems,” in Lecture Notes in Computer Science - ICCS 2022, p. 202-217, Springer International Publishing, 2022

V. Gilbert, J. Rodriguez, S. Louise, and R. Sirdey, “Solving higher order binary optimization problems on nisq devices: experiments and limitations,” in Lecture Notes in Computer Science - ICCS 2023, p.224-232, Springer Nature Switzerland, 2023

V. Gilbert, S. Louise, and R. Sirdey, “Taqos: A benchmark protocol for quantum optimization systems,” in Lecture Notes in Computer Science – ICCS 2023, p.168-176, Springer Nature Switzerland, 2023

V. Gilbert, and S. Louise, “Quantum annealers chain strengths: A simple heuristic to set them all,” in Lecture Notes in Computer Science - ICCS 2024, p.292-306, Springer Nature Switzerland, 2024

V. Gilbert, J. Rodriguez, and S. Louise “Benchmarking quantum annealers with near-optimal minor-embedded instances,” in 2024 IEEE International Conference on Quantum Computing and Engineering (QCE), IEEE, 2024

References

[AL18] Albash, T., & Lidar, D. A. (2018). Adiabatic quantum computation. Reviews of Modern Physics, 90(1), 015002.

[PCL+21] Y. Pang, C. Coffrin, A. Y. Lokhov, and M. Vuffray, “The potential of quantum annealing for rapid solution structure identification,” Constraints, vol. 26, no. 1, pp. 1–25, 2021.

[TAM+22] B. Tasseff, T. Albash, Z. Morrell, et al., “On the emerging potential of quantum annealing hardware for combinatorial optimization,” arXiv preprint arXiv:2210.04291, 2022

[LCG+24] T. Lubinski, C. Coffrin, C. McGeoch, P. Sathe, et al., “Optimization applications as quantum performance benchmarks,” ACM Transactions on Quantum Computing, vol. 5, no. 3, pp. 1–44, 2024.

[KNR+24] A. D. King, A. Nocera, et al., “Computational supremacy in quantum simulation,” arXiv preprint arXiv:2403.00910, 2024.

[RS95] N. Robertson and P. Seymour, “Graph minors .XIII. the disjoint paths problem,” Journal of Combinatorial Theory, Series B, vol. 63, pp. 65–110, Jan. 1995.

[Cho11] V. Choi, “Minor-embedding in adiabatic quantum computation: Ii. Minor-universal graph design,” Quantum Information Processing, vol. 10, no. 3, pp. 343–353, 2011

[RWJ+14] T. F. Rønnow, Z. Wang, J. Job, S. Boixo, S. V. Isakov, et al., “Defining and detecting quantum speedup,” science, vol. 345, no. 6195, pp. 420–424, 2014.

References

[VMK+14] D. Venturelli, S. Mandrà, S. Knysh, B. O'Gorman, R. Biswas, and V. Smelyanskiy, "Quantum optimization of fully connected spin glasses," Physical Review X, vol. 5, no. 3, p. 031040, 2015.

[Cho08] V. Choi, "Minor-embedding in adiabatic quantum computation: I. the parameter setting problem," Quantum Information Processing, vol. 7, pp. 193–209, 2008

[HIM+18] Hamerly, R., Inagaki, T., McMahon, et al.: Experimental investigation of performance differences between coherent ising machines and a quantum annealer. Science Advances 5(5) (2019)

[WWC+22] Willsch, D., Willsch, M., et al.: Benchmarking advantage and d-wave 2000q quantum annealers with exact cover problems. QIP 21(4) (2022)

[Dji23] Djidjev, H. N. (2023). Logical qubit implementation for quantum annealing: augmented Lagrangian approach. Quantum Science and Technology, 8(3), 035013.

[MCD20] Moussa, C., Calandra, H., & Dunjko, V. (2020). To quantum or not to quantum: towards algorithm selection in near-term quantum optimization. Quantum Science and Technology, 5(4), 044009.

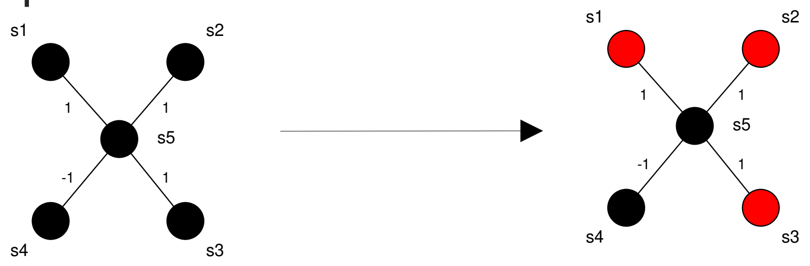
[SM23] Smith-Miles, K., & Muñoz, M. A. (2023). Instance space analysis for algorithm testing: Methodology and software tools. ACM Computing Surveys, 55(12), 1-31.

[Pre21] Preskill, J. (2023). Quantum computing 40 years later. In Feynman Lectures on Computation (pp. 193-244). CRC Press.

4. Appendix

Selection of best Annealing time

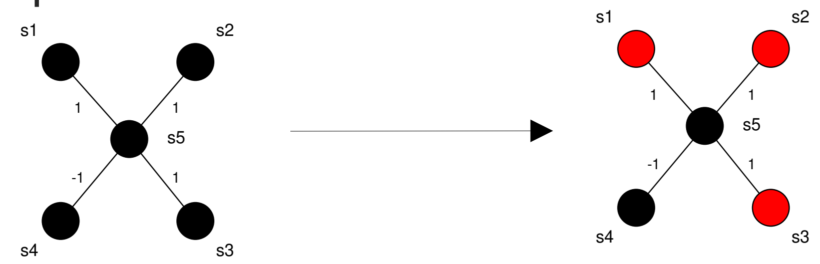
- Quality metrics are problem-dependent
  - Max-cut problem: The cut size



cut-size = 3

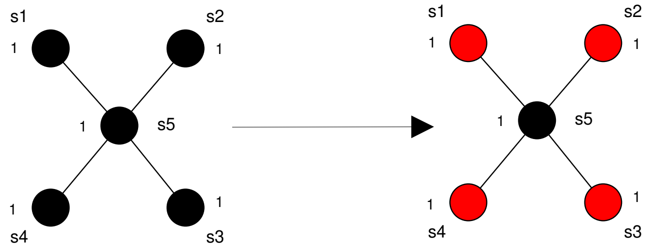
Selection of best Annealing time

- Quality metrics are problem-dependent
  - Max-cut problem: The cut size



cut-size = 3

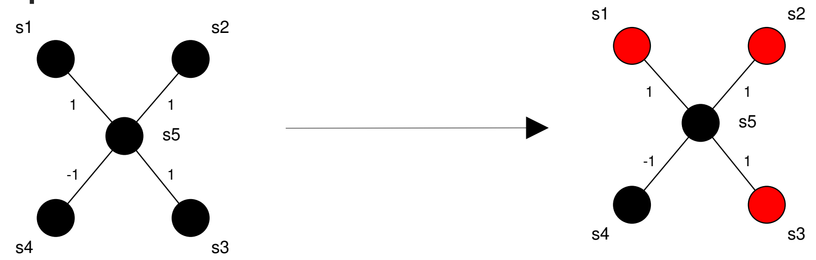
  - Maximum Independent set problem: The size of the independent set.



IS size = 4

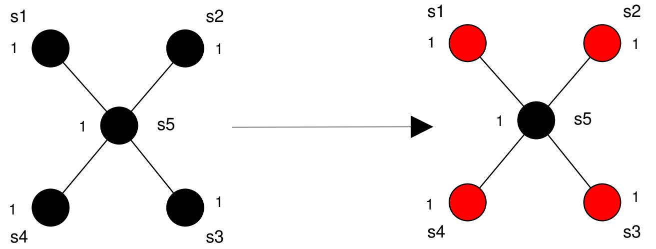
Selection of best Annealing time

- Quality metrics are problem-dependent
  - Max-cut problem: The cut size



cut-size = 3

  - Maximum Independent set problem: The size of the independent set.

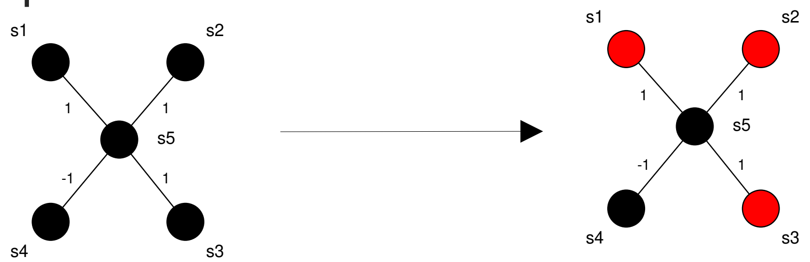


IS size = 4

- Time measurement
  - 60s time window for branch & bound algorithm
  - 1s time window for Tabu Search (C implementation)
  - 1s time window for D-Wave Q

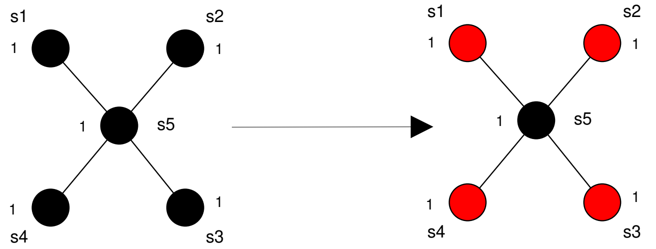
Selection of best Annealing time

- Quality metrics are problem-dependent
  - Max-cut problem: The cut size



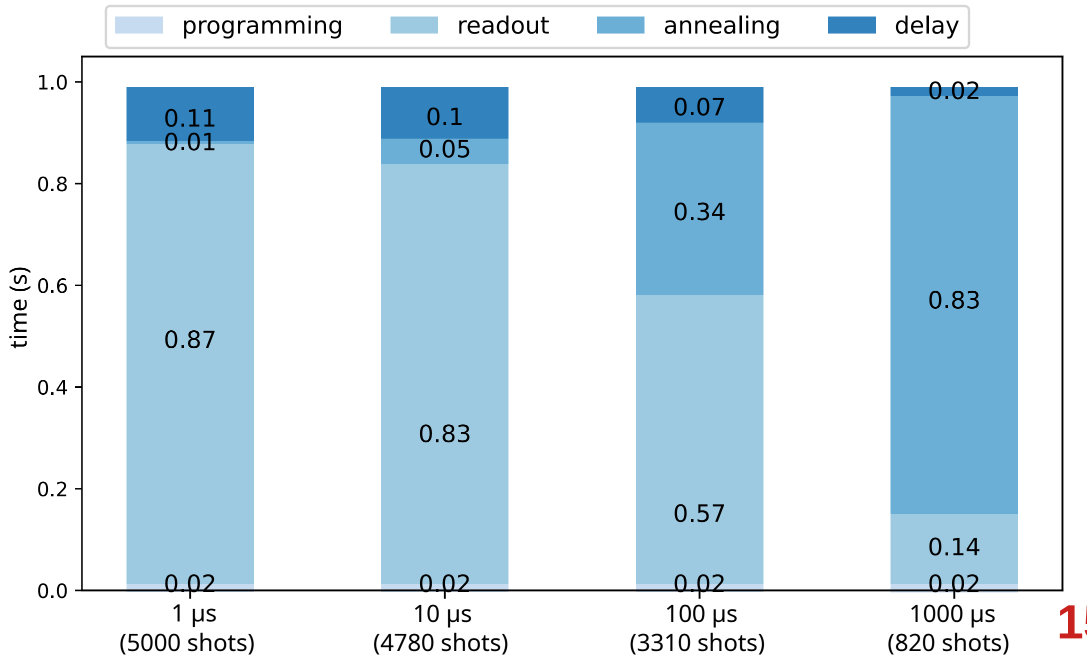
cut-size = 3

  - Maximum Independent set problem: The size of the independent set.



IS size = 4

- Time measurement
  - 60s time window for branch & bound algorithm
  - 1s time window for Tabu Search (C implementation)
  - 1s time window for D-Wave Q



Optimal mapping

- How to assess the quality of a mapping:
  - D-Wave QA has a regular topology

    For Pegasus topology: $c_{\text{phys}} = 15$

Optimal mapping

- How to assess the quality of a mapping:
  - D-Wave QA has a regular topology

    For Pegasus topology: $c_{\text{phys}} = 15$

  - Definition of a lower bound on the number of physical qubits used in a mapping:

$$n_{\phi(v)^*} = \begin{cases} 1 \text{ if } deg(v) \leq c_{\text{phys}} \\ 2 \text{ if } c_{\text{phys}} < deg(v) \leq (2c_{\text{phys}} - 2) \\ \left\lceil \frac{deg(v) - (2c_{\text{phys}} - 2)}{c_{\text{phys}} - 2} \right\rceil + 2 \text{ otherwise} \end{cases}$$

Optimal mapping

- How to assess the quality of a mapping:
  - D-Wave QA has a regular topology

    For Pegasus topology: $c_{\text{phys}} = 15$

  - Definition of a lower bound on the number of physical qubits used in a mapping:

$$n_{\phi(v)^*} = \begin{cases} 1 \text{ if } deg(v) \leq c_{\text{phys}} \\ 2 \text{ if } c_{\text{phys}} < deg(v) \leq (2c_{\text{phys}} - 2) \\ \left\lceil \frac{deg(v) - (2c_{\text{phys}} - 2)}{c_{\text{phys}} - 2} \right\rceil + 2 \text{ otherwise} \end{cases}$$

  - Compute the overhead ratio considering this bound:

$$r_{\text{o}} = \frac{n_{\phi}}{\sum_{v \in V_s} n_{\phi(v)^*}}$$

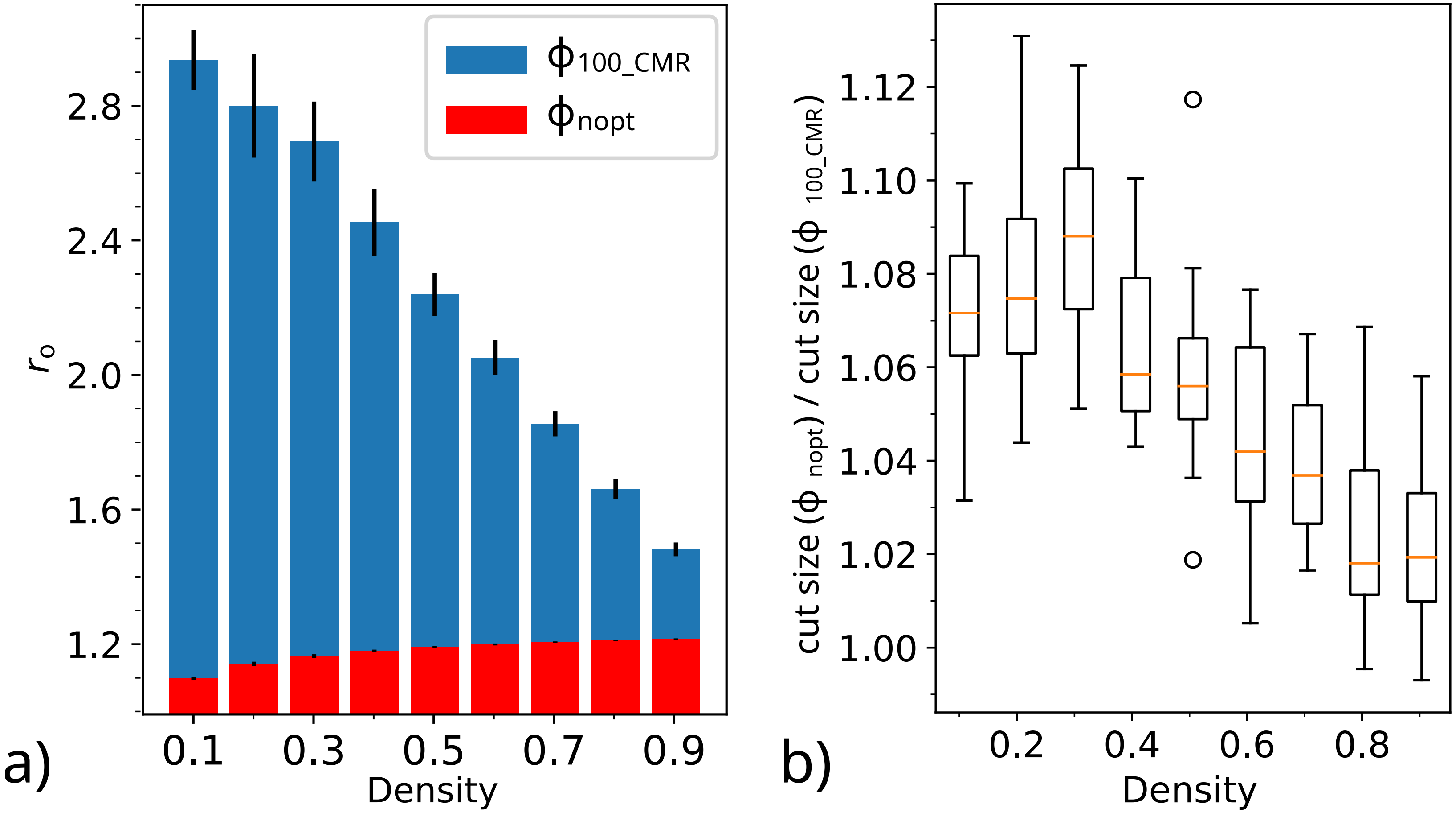
Optimal mapping

- Comparison of the performance of our generation method against state of the art embedding method

at $d = 1$

$|V_s| = 100$

$|V_t| = 982$



<u>Assumption:</u> Instances with less duplicated qubits are more easily solved by QA => Seems to be true

III- Shape of the logical qubit